

Math 127: Review for Final Exam

Study Tips:

Begin studying for the final by redoing all of your old tests. Try to do the problems without looking at your old work or the answer key. Put a star by any problem that you can't remember how to do. The starred problems will tell you where you need to focus your studying. Go to the sections the starred problems came from and practice those problems. Come see me or go to the Math Lab next week for help if you get stuck!

I. Solving Equations and Inequalities

1. Rational equations: Solving equations containing fractions: Find the LCD and clear all fractions by multiplying the equation (each fraction or term) by the LCD. CHECK YOUR ANSWER FOR DIVISION BY ZERO!!!

2. Absolute value equations: KEY POINT: when you remove the absolute value symbol, the equation splits into two. You also MUST check your answer in the original equation.

$$|x| = a \Rightarrow x = a \quad -x = a$$

3. Absolute value inequalities: KEY POINT: when you remove the absolute value symbol, the inequality splits into two.

$$|x| < a \Rightarrow x < a \text{ AND } -x < a \quad \text{Remember: Less Than AND}$$

$$|x| > a \Rightarrow x > a \text{ OR } -x > a \quad \text{Remember: Great OR}$$

4. Solve a quadratic equation by

- factoring (set equation = 0 first, THEN factor)
- taking square roots (remember to take + / - the square root of the right side)
- completing the square (divide coefficient of x by 2, square it, add to both sides of equation)
- quadratic formula (memorize this formula)

5. Recognize and solve equations that are quadratic in form

Identify: 3 terms and leading power = 2. • middle power

Solve: Set u = middle term base and power. BE SURE TO RESUBSTITUTE at the end (solve for the ORIGINAL variable).

6. Solve log equations: Consolidate logs into single log, then rewrite into exponential form

7. Solve exponential equations: Isolate exponential portion then take log or ln of both sides of equation.

8. Three-variable systems of linear equations

Method of solution: Elimination KEY POINT: Use elimination on two different pairs of equations to create two new equations which have only 2 variables. ELIMINATE THE SAME VARIABLE FROM EACH PAIR! Solve this new two variable system.

EXTRANEIOUS SOLUTIONS: Know which types of equations can have extraneous solutions

- Equations with fractions
- Absolute value equations
- Equations with square roots (or any even roots)
- Log equations

II. Applications:

1. Geometry problems: Sketch a picture and label every part of it. List all given information.

2. Motion problems: Rate x Time = Distance (set up an RTD table).

3. Variation:

y varies directly as x $\Rightarrow y = kx$, y varies inversely as x $\Rightarrow y = k/x$, y varies jointly as x and z $\Rightarrow y = k \cdot x \cdot z$.

Plan of attack: 1. Translate variation statement into a formula 2. Plug in related values of variables to find k, 3. Use k and OTHER values of variables to find the unknown variable.

4. Work (job) problems: $\frac{\text{time together}}{\text{time alone}} + \frac{\text{time together}}{\text{time alone}} = 1$ (job)

5. Projectile motion problems: Find when an object hits the ground or reaches its maximum height.

III. Functions

1. Definition of a Function: A set of ordered pairs for which each element of the domain (1 input) corresponds to exactly one element of the range (1 output). Know this definition and be able to identify a function. Specifically,

- Given a graph, use the Vertical Line Test;
- Given a table of data (or set of (x,y) pairs) look for 2 repeated x's paired with different y's

2. Domain: the set of all of the input values, i.e., x-values. To find

- Given graph, sweep through the graph, left-to-right
- Given a table of data (or set of (x,y) pairs), domain is the set of the x-values
- Given an equation, look for problematic x-values that can cause division by zero. Then the domain is all real numbers EXCEPT these values that cause problems.

3. Range: the set of all of the output values, i.e., y-values

- Given a graph, sweep through the graph, bottom-to-top
- Given a table of data (or set of (x,y) pairs), domain is the set of the y-values

4. Evaluating a function and Composition of functions: To find $f(\text{number or expression})$, replace all x's with the number or expression. Use this technique when finding the composition of two functions. $(f \circ g)(x) = f(g(x))$

5. Operations with functions: Be able to add, subtract, multiply and divide functions using a table of data, a graph or the function equations.

6. Find the inverse of a function from a table of data, a graph or an equation (KEY IDEA: switch x and y)

7. Determine whether a given function is one-to-one (Each x, y pair is unique...no doubling of y's, use the Horizontal Line Test)

IV. Lines and Graphing

1. Write the equation of a line: As soon as you see the words "FIND THE EQUATION OF THE LINE ..." write down the point-slope equation: $y - y_1 = m(x - x_1)$. Find the slope if necessary, then substitute a point for (x_1, y_1) and the slope for m.

2. Slope of a line: Remember that slope = rise / run

a. given two points: $m = \frac{y_2 - y_1}{x_2 - x_1}$

- b. given a point and an equation: solve the equation for y to get the line equation $y = mx + b$. Slope = x coefficient
parallel lines: same slope
perpendicular lines: one slope is opposite in sign and reciprocal of the other

3. Horizontal lines: Equation is of the form $y = \text{some constant}$ (e.g., $y = 1$). SLOPE IS 0.

4. Vertical lines: Equation is of the form $x = \text{some constant}$ (e.g. $x = 2$). SLOPE IS UNDEFINED.

5. Quadratic Functions and Parabolas:

- Put a quadratic function into standard form $y = a(x - h)^2 + k$ by completing the square.
- Graph the function by identifying the vertex (h,k) and two other points. Include the axis of symmetry, the x-intercept(s) if they exist and the y-intercept.
- Determine the orientation (opening up or down) of a parabola based on the sign of a

6. Graph exponential functions (also recognize the graph if it's given to you)

7. Graph log functions (also recognize the graph if it's given to you)

8. Graph the solution to Linear Inequalities in two variables:

KEY POINT: Graph the equations(s), using dashed graph for $<, >$ solid for \leq, \geq , then check a point in the inequality. Mark overlapping shaded areas as "SOLUTION" for a system.

V. Sequences and Series

1. Given a rule for a_n , be able to find any term in a sequence.
2. Given a series $\sum_{i=start}^{end} a_i$, be able to expand and evaluate (find the sum).
3. Given a binomial $(x + y)^n$, be able to expand it using Pascal's triangle to find the coefficients. When x and y are other values, e.g., $(2a - 3)^n$, be able to expand and simplify the terms.

VI. Miscellaneous

1. Rational exponents: Remember that a negative power inverts the expression

$$x^{\frac{power}{root}} = \left(\sqrt[root]{x}\right)^{power}$$

2. Complex numbers:
 - a. rewrite $\sqrt{-number}$ in terms of i
 - b. Add, subtract, multiply complex numbers (treat i like x, but remember to substitute $i^2 = -1$)
 - c. Divide complex numbers by rationalizing the denominator (multiply by the conjugate)
3. Manipulate log expressions, using the properties of logs.
4. Simplify Complex Fractions:
Combine and Divide: Combine component fractions in the top and bottom of the large fraction then treat as a division problem