

Math 127: Test 2
(Exponents, Division, Quadratic Equations, Complex Numbers)

Name: KEY

Please show all of your work in neat, clear steps. Credit will be based on the amount of correct work shown. You must use the method specified in the problem in order to receive credit for the answer; i.e., the credit for the problem is based on the method, NOT on the answer! Each problem is worth 5 points unless otherwise noted.

1. Divide, using synthetic division: $\frac{2x^3 - 5x - 4}{x-1} = \frac{2x^3 + 0x^2 - 5x - 4}{x-1}$

$$\begin{array}{r|rrrr} 1 & 2 & 0 & -5 & -4 \\ & & 2 & 2 & -3 \\ \hline & 2 & 2 & -3 & -7 \end{array}$$

$$= \boxed{2x^2 + 2x - 3 + \frac{-7}{x-1}}$$

- Change sign
- Missing coef
- Interpret

2. Divide, using long division: $\frac{x^3 - 2x^2 + 3x - 1}{x^2 + 5x - 1} = \boxed{x - 7 + \frac{39x - 8}{x^2 + 5x - 1}}$

$$\begin{array}{r} x^2 + 5x - 1 \overline{) x^3 - 2x^2 + 3x - 1} \\ \underline{-(x^3 + 5x^2 - x)} \\ -7x^2 + 4x - 1 \\ \underline{-(-7x^2 - 35x + 7)} \\ 39x - 8 \end{array}$$

Sign error: 1 pt per

3. (3 pts) Circle the division problem that CAN'T be done using synthetic division. Don't actually do the division!

a) $\frac{5x^3 - 2x^2 + 3x - 4}{x-1}$

b) $\frac{x^3 - 1}{x+2}$

c) $\frac{x^3 - 3x - 7}{x^2 - 1}$

4. (6 pts) Simplify all of the following expressions. Your answer should have no negative exponents.

(a) $x^4 \cdot (2x^2)^{-3}$
 $= x^4 \cdot 2^{-3} x^{-6}$
 $= \frac{x^4}{2^3 x^6} = \boxed{\frac{1}{2^3 x^2}}$ OR $\boxed{\frac{1}{8x^2}}$

(b) $\left(\frac{3x^{-3}}{x^5}\right)^{-2} = \frac{3^{-2} x^6}{x^{-10}}$
 $= \frac{x^6 x^{10}}{3^2} = \boxed{\frac{x^{16}}{3^2}}$ OR $\boxed{\frac{x^{16}}{9}}$

5. (6 pts) Rewrite each of the following expressions as a radical then evaluate WITHOUT using a calculator. You must show work for credit.

(a) $8^{\frac{2}{3}}$
 $= \left(\sqrt[3]{8}\right)^2$
 $= 2^2$
 $= \boxed{4}$

(b) $9^{\frac{1}{2}}$ $= \frac{1}{9^{\frac{1}{2}}} = \frac{1}{\sqrt{9}} = \boxed{\frac{1}{3}}$
 OR
 $\sqrt{9^{-1}} = \sqrt{\frac{1}{9}} = \boxed{\frac{1}{3}}$

6. Rewrite each of the following square roots in terms of "i" and simplify as much as possible.

a) $\sqrt{-25}$
 $= \boxed{5i}$

b) $\sqrt{-12}$
 $= \boxed{2i\sqrt{3}}$

c) $\frac{6 + \sqrt{-16}}{4}$
 $= \frac{6 + 4i}{4} = \frac{3 + 2i}{2}$
 $= \boxed{\frac{3}{2} + i}$

7. Simplify the following expression. Express your answer in the form $a + bi$.

$(1 + 2i)(4 - 3i) + 5i(1 - 4i)$

$4 - 3i + 8i - 6i^2 + 5i - 20i^2$
 $= 4 + 6 + 20 + -3i + 8i + 5i$
 $= \boxed{30 + 10i}$

$i^2 = -1$

8. (a) What is the conjugate of the complex number $3 + 4i$? $3 - 4i$

(b) "Divide". Express your final answer in the form $a + bi$.

$$\frac{5}{(3+4i)(3-4i)} = \frac{15 - 20i}{9 - 12i + 12i - 16i^2} = \frac{15 - 20i}{9 + 16} = \frac{15 - 20i}{25}$$
$$= \frac{15}{25} - \frac{20}{25}i = \boxed{\frac{3}{5} - \frac{4}{5}i}$$

9. (12 pts) Factor the following polynomials COMPLETELY. If it isn't factorable, then write "prime".

a) $5y^3 - 30y^2 - 80y^2$

$$= 5y^2(y^2 - 6y - 16)$$
$$= \boxed{5y^2(y - 8)(y + 2)}$$

b) $x^2 + 25$

$$\boxed{\text{prime}}$$

c) $2x^4 - 32$

$$= 2(x^4 - 16)$$
$$= 2(x^2 + 4)(x^2 - 4)$$
$$= \boxed{2(x^2 + 4)(x + 2)(x - 2)}$$

d) $12y^3 - 12y^2 - 45y$

$$= 3y(4y^2 - 4y - 15)$$
$$= \boxed{3y(2x + 3)(2x - 5)}$$

10. Solve the equation by Square Root Method.

$$4x^2 - 13 = 0$$

$$\frac{4x^2}{4} = \frac{13}{4}$$

$$x^2 = \frac{13}{4}$$

$$\rightarrow \sqrt{x^2} = \pm \sqrt{\frac{13}{4}}$$

$$\boxed{x = \pm \frac{\sqrt{13}}{2}}$$

11. Complete the square and factor the following expressions:

a) $x^2 - 6x + 9 = (x - 3)^2$

b) $x^2 + 7x + \frac{49}{4} = (x + \frac{7}{2})^2$

① $\frac{1}{2}(-6) = -3$ ② $(-3)^2 = 9$

① $\frac{1}{2} \cdot 7 = \frac{7}{2}$ ② $(\frac{7}{2})^2 = \frac{49}{4}$

12. Solve the following equation by completing the square. Simplify your answer as much as possible.

$$\frac{2x^2 = -8x + 6}{2}$$

$$x^2 = -4x + 3$$

$$x^2 + 4x + \frac{4}{4} = 3 + \frac{4}{4}$$

$$(x+2)^2 = 7$$

$$x+2 = \pm\sqrt{7}$$

$$x = -2 \pm \sqrt{7}$$

13. Solve each of the equation by using the Quadratic Formula. Simplify your answer as much as possible.

$$2x^2 = -8x + 6$$

$$2x^2 + 8x - 6 = 0$$

a b c

→ note: you can divide the equation by 2 at the beginning (makes the numbers more manageable in the formula!)

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(2)(-6)}}{2(2)}$$

$$\frac{64}{48} = \frac{-8 \pm \sqrt{112}}{4} = \frac{-8 \pm 4\sqrt{7}}{4} = \boxed{-2 \pm \sqrt{7}}$$

14. (a) Find the value of the Discriminant for the following equation: $3x^2 - 2x + 6 = 0$

$$b^2 - 4ac = (-2)^2 - 4(3)(6) = 4 - 72 = \boxed{-68}$$

(b) Does the equation have real number solutions? How can you tell WITHOUT SOLVING the equation?

No, because the discriminant is negative.

15. (10 pts) The following equation is Quadratic in Form. Solve, using the Method of u-Substitution.

$$x^{\frac{2}{3}} - 7x^{\frac{1}{3}} + 12 = 0$$

$$u = x^{\frac{1}{3}}$$

$$u^2 - 7u + 12 = 0$$

$$(u-3)(u-4) = 0$$

$$u-3=0 \quad u-4=0$$

$$u=3$$

$$u=4$$

$$u=3$$

↓

$$x^{\frac{1}{3}} = 3$$

$$(x^{\frac{1}{3}})^3 = (3)^3$$

$$x = 27$$

$$u=4$$

↓

$$x^{\frac{1}{3}} = 4$$

$$(x^{\frac{1}{3}})^3 = (4)^3$$

$$x = 64$$

16. Circle the equation that is NOT Quadratic in Form. Do NOT solve the equations!

(a) $x^6 - 7x^3 + 12 = 0$

(b) $x^{\frac{1}{6}} - 7x^{\frac{1}{2}} + 12 = 0$

(c) $x^3 - 7x^2 + 12 = 0$

17. A ball is thrown into the air at a speed of 64 ft/sec. The equation that gives the height, h , in feet, of the ball above the ground after t seconds is: $h = -16t^2 + 64t + 192$.

When will the ball be on the ground? Use algebra to solve this problem, not guess-and-check.

$$0 = -16t^2 + 64t + 192$$

$$0 = -16(t^2 - 4t - 12)$$

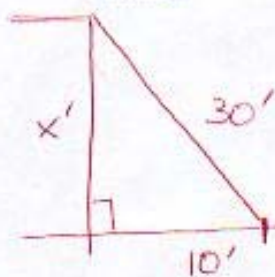
$$0 = -16(t-6)(t+2)$$

$$t = 6 \quad t = -2$$

not realistic

The ball will be on the ground in 6 seconds

18. The base of a 30 foot ladder is 10 feet from the building. If the ladder reaches the flat roof, how tall is the building?



$$a^2 + b^2 = c^2$$

$$x^2 + 10^2 = 30^2$$

$$x^2 + 100 = 900$$

$$x^2 = 800$$

$$x = \sqrt{800}$$

$$x = \sqrt{800} \text{ feet}$$

OR

20 $\sqrt{2}$ feet is the height of the building

Extra Credit. Beginning with $ax^2 + bx + c = 0$, derive the Quadratic Formula by completing the square. (5 points) (Do the derivation on the back of this page.)