#19, Round to 4 decimal places

#20 Change 7o million to 7 million

Math 127: Test # 4: **Name:** KEY

Please organize your work in a clear, logical manner. Credit for each problem is based on the amount of correct work shown. Simplify all of your answers as much as possible.

1. (8 pts) Given $f(x) = x^2 + 3x + 1$ and $g(x) = x - 2$, do the following (simplify your answers):

(a) Find $(g \circ f)(5)$

$= g(f(5))$

$= g(5^2 + 3(5) + 1)$

$= g(41)$

$= 41 - 2 = \boxed{39}$

(b) Find $(g \circ f)(x)$

$= g(f(x))$

$= (x^2 + 3x + 1) - 2$

$= \boxed{x^2 + 3x - 1}$

(c) Find $(f - g)(x)$

$= f(x) - g(x)$

$= x^2 + 3x + 1 - (x - 2)$

$= \boxed{x^2 + 2x + 3}$

(d) Find $(f \circ g)(x)$

$= f(g(x))$

$= (x-2)^2 + 3(x-2) + 1$

$= x^2 - 4x + 4 + 3x - 6 + 1$

$= \boxed{x^2 - x - 1}$

2. (6 pts) Use the given data tables to find the following:

x	0	1	2	3
f(x)	-3	3	1	7

x	0	1	2	3
g(x)	10	8	6	4

(a) $(f \cdot g)(3)$

$= f(3) \cdot g(3)$

$= 7 \cdot 4 = \boxed{28}$

(b) $\left(\frac{g}{f}\right)(0)$

$= \frac{g(0)}{f(0)} = \frac{10}{-3} = \boxed{-\frac{10}{3}}$

(c) $g(f(1)) = g(3)$

$= \boxed{4}$

3. (4 pts) Determine the domain of the function. Write your answer in set-builder notation.

(a) $f(x) = \frac{3x}{x-5}$

Domain: $\{x \mid x \neq 5\}$

4. (5 pts) Find the inverse of the function $f(x) = 3x - 6$. Write your final answer using "f inverse" notation.

$y = 3x - 6$

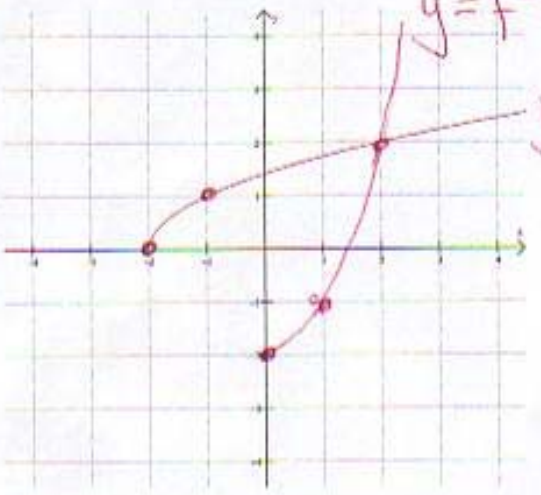
inverse $x = 3y - 6$

$x + 6 = 3y$

$y = \frac{x+6}{3}$

$f^{-1}(x) = \frac{x+6}{3}$

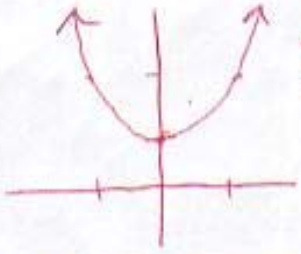
5. (4 pts) Use the given graph of f to graph f^{-1} (draw f^{-1} on the given axes)



x	f(x)
−2	0
−1	1
2	2

x	$f^{-1}(x)$
0	−2
1	−1
2	2

6. (3 pts) Is the function $f(x) = x^2 + 1$ one-to-one? How can you tell? (Hint: sketch a graph).



Fails Horizontal Line Test so f is NOT one-to-one

7. (3 pts) Put the log expression into exponential form:

$\log_5(25) = 2$ $\quad 5^2 = 25$

8. (3 pts) Put the exponential expression into log form:

$a^y = x$ $\quad \log_a(x) = y$

9. (6 pts) Combine into a single log:
$\log_4(a) + 2\log_4(b) - \frac{1}{4}\log_4(c)$

$= \log_4(a) + \log_4 b^2 - \log_4 c^{1/4}$

$= \log_4\left(\frac{a \cdot b^2}{c^{1/4}}\right)$ OR $\log_4\left(\frac{ab^2}{\sqrt[4]{c}}\right)$

10. (6 pts) Expand the expression, i.e., write as the sum, difference and multiples of logs, using the properties of logs. There should be no exponents or square roots in your final answer.

$\log\left(\frac{x^3 z}{\sqrt{y}}\right) = \log x^3 + \log z - \log \sqrt{y}$

$= 3\log x + \log z - \frac{1}{2}\log y$

11. (6 pts) Evaluate without using a calculator (no credit will be given for calculator answers). If the value is undefined, then say so.

a.) $\log_3(9) =$ 2 because $3^2 = 9$

b.) $\log_2\left(\frac{1}{16}\right) =$ -4 because $2^{-4} = \frac{1}{16}$

c.) $\log_5(\sqrt{5}) =$ $\frac{1}{2}$ because $5^{1/2} = \sqrt{5}$

d.) $\log_3(-3) =$ undef.

e.) $\log_2(0) =$ undef.

f.) $\log_8(1) =$ 0 because $8^0 = 1$

12. (4 pts) Find the following without using a calculator:

a.) $e^{\ln(15)} =$ 15

b.) $\log(10^{13}) =$ 13

c.) $\ln(e^5) =$ 5

d.) $2^{\log_2(5x+10)} =$ $5x+10$

13. (4 pts) Use your calculator to evaluate the following expressions (give your answer to 4 decimal places).

(a) $\log(107) \cong$ 2.0294

(b) $\ln(32.5) \cong$ 3.4812

(c) $\frac{1+\ln 2}{\ln 5} \cong$ 1.0520

(d) $e^{1.06} \cong$ 2.8864

14. (8 pts) Solve for x:

(a) $\log_3(x) = -2$ (8pts)

$3^{-2} = x$

$x = \frac{1}{3^2} = \frac{1}{9}$

$\boxed{x = \frac{1}{9}}$

(b) $\log_2(x) + \log_2(x-6) = 4$ (8pts)

$\log_2(x^2 - 6x) = 4$

$2^4 = x^2 - 6x$

$x^2 - 6x - 16 = 0$

$(x-8)(x+2) = 0$

$\boxed{x = 8}$ $\boxed{x = -2}$ extraneous

15. (8 pts) Solve for x. Leave your answer in exact terms, i.e., you don't need to use a calculator for this problem.

(a) $2^{3x} = 15$

$\ln 2^{3x} = \ln 15$

$3x \cdot \ln 2 = \ln 15$

$\frac{3x \ln 2}{3 \ln 2} = \frac{\ln 15}{3 \ln 2}$

$\boxed{x = \frac{\ln 15}{3 \ln 2}}$

(b) $2000 = 1000e^{.05t}$

$\frac{2000}{1000} = \frac{1000e^{.05t}}{1000}$

$2 = e^{.05t}$

$\ln 2 = \ln e^{.05t}$

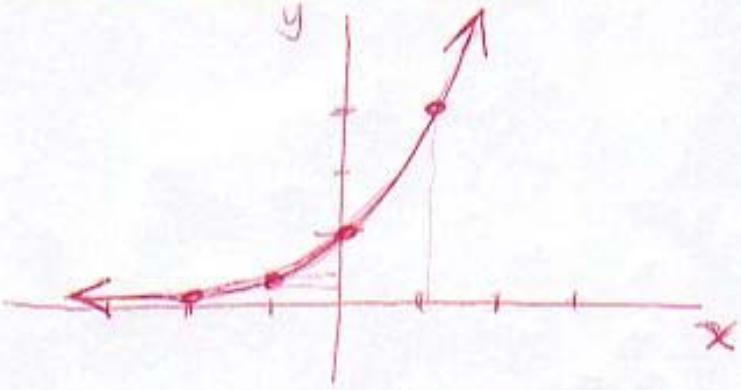
$\ln 2 = .05t \ln e$ (1)

$\frac{\ln 2}{.05} = \frac{.05t}{.05}$

$\boxed{t = \frac{\ln 2}{.05}}$

16. (6 pts) Graph $f(x) = 3^x$. Plot at least 4 points, including some with NEGATIVE x VALUES!

x	y
-2	$3^{-2} = 1/9$
-1	$3^{-1} = 1/3$
0	$3^0 = 1$
1	$3^1 = 3$
2	$3^2 = 9$

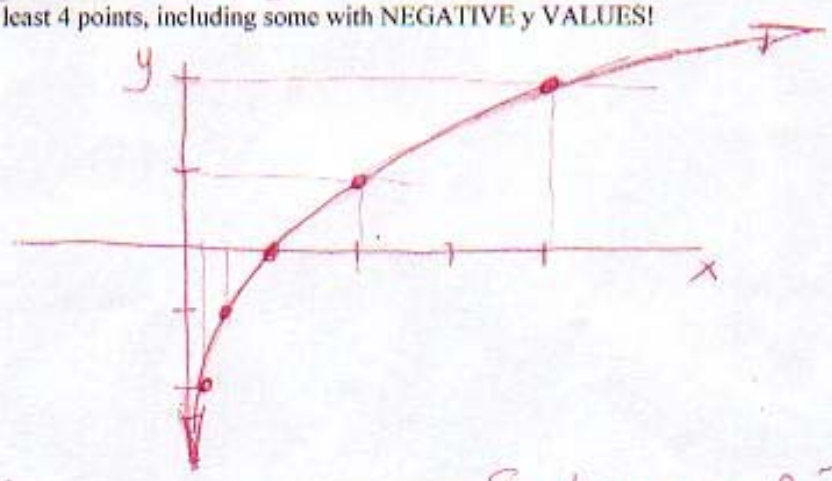


What is the domain of $f(x) = 3^x$? all reals $\{x \mid x \text{ is real}\}$ What is the range? $\{y \mid y > 0\}$

17. (6 pts) Graph $y = \log_2(x)$. Plot at least 4 points, including some with NEGATIVE y VALUES!

$2^y = x$

x	y
$2^{-2} = 1/4$	-2
$2^{-1} = 1/2$	-1
$2^0 = 1$	0
$2^1 = 2$	1
$2^2 = 4$	2



What is the domain of $y = \log_2(x)$? $\{x \mid x > 0\}$ What is the range? $\{y \mid y \text{ is real}\}$

18. (4 pts) The compound interest formula is $A = P\left(1+\frac{r}{n}\right)^{nt}$. Determine how much would be in a savings account after 3 years if $1000 is deposited and the interest rate is 3%, compounded monthly. Round your answer to the nearest cent.

$A = ?$
$P = \$1{,}000$
$t = 3$
$r = .03$
$n = 12$

$A = 1000\left(1+\frac{.03}{12}\right)^{12(3)}$

$= 1000(1.0025)^{36}$

$\cong 1094.05$

There will be $1094.05 in the account.

19. (6 pts) After 6 hours, 10 mg of a certain radioactive element decays to be 5 mg. What is the decay rate, k, for this element? $A = A_0e^{-kt}$

$A_0 = 10$
$A = 5$
$t = 6$
$k = ?$

$5 = 10e^{-k(6)}$

$\frac{5}{10} = \frac{10e^{-6k}}{10}$

$\frac{1}{2} = e^{-6k}$

$\ln\left(\frac{1}{2}\right) = \ln e^{-6k}$

$\ln\left(\frac{1}{2}\right) = \frac{-6k}{-6}\cdot \ln e$

$\frac{\ln\left(\frac{1}{2}\right)}{-6} = k$

$k \cong .1155$

20. Extra credit (5 points): If the half-life of a certain radioactive element used for dating rocks is 10 million years, what percentage of the element would you expect to find in a rock that is 7 million years old?

Half life: $t = 10$ million years
$A = \frac{1}{2}A_0$

$\frac{1}{2}A_0 = A_0e^{-k(10)}$

$\frac{1}{2} = e^{-10k}$

$\ln\left(\frac{1}{2}\right) = \ln e^{-10k}$

$k = \ln\left(\frac{1}{2}\right)/-10$

$\cong .0693$

So for $t = 7$ we have

$A = A_0e^{-.0693(7)}$

$= A_0(.6156)$

So 62% remains