Math 229: Combinations of Functions (Supplement for Chapter 8)

General Sine Function (with no phase shift) $y = A\sin(Bx) + D$

Describe how each of the following constants affects the parent graph of $y = \sin x$

What does A control?

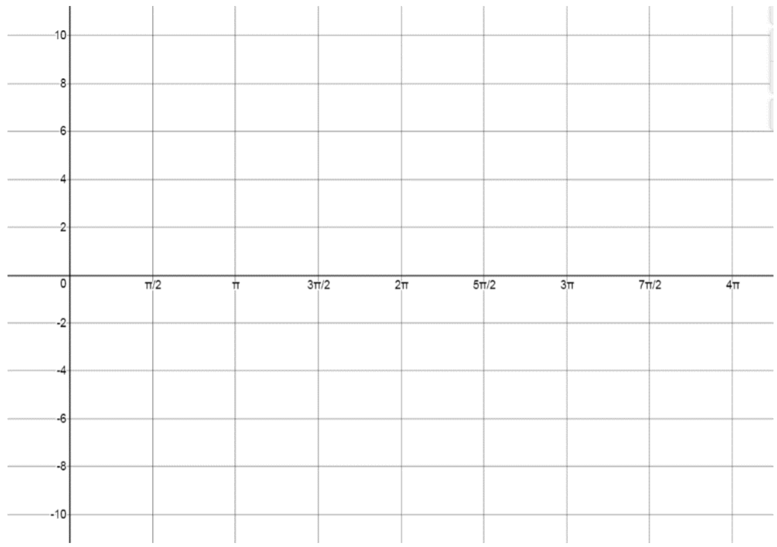
What does B control?

What does D control?

Variable Amplitude:

Normally A is a constant.

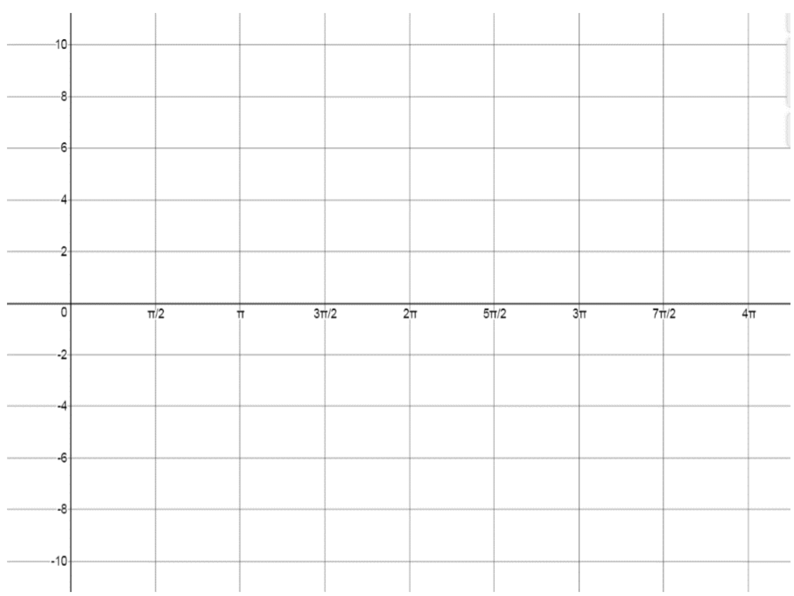
Review! Make a quick sketch of two periods
(positive x values) of $y = 2\sin(x)$
Be sure to dash in an "Amplitude Envelope" at y = 2 and y = -2.



What happens if A is not constant, i.e., $A = A(x)$?

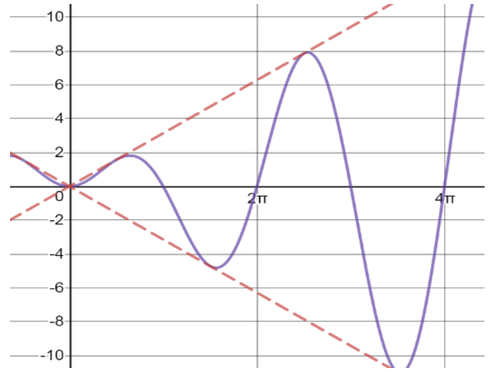
Example: $y = x\sin(x)$
What is "A" for this function?

Predict what effect this new kind of "A" (variable) will have on the graph:



By hand, sketch y = x and y = -x as an "Amplitude Envelope" and fill in the sine graph. Remember that the amplitude doesn't affect the zeros (midline points), so these will be the same as for a regular sine graph.

Now graph $y = x\sin(x)$ using Desmos and see if your prediction was correct. Add the graphs of y = x and y = -x in your Desmos graph.



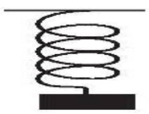
Note: The new function $y = x\sin(x)$ is NOT periodic since it doesn't return to the same values of y as it cycles through its oscillations.

However, it has a distinct pattern that is important in real applications! Specifically…

Damped Oscillations: In the real world, will a spring keep bouncing up and down forever as time goes on? ________

If not, what would the graph of the position of a weight on the end of bouncing spring look like, over time? Sketch a graph next to the picture of the weight, showing its displacement as a function of time.

Compressed spring



Equilibrium: -

Dash in the amplitude envelope on your graph above.

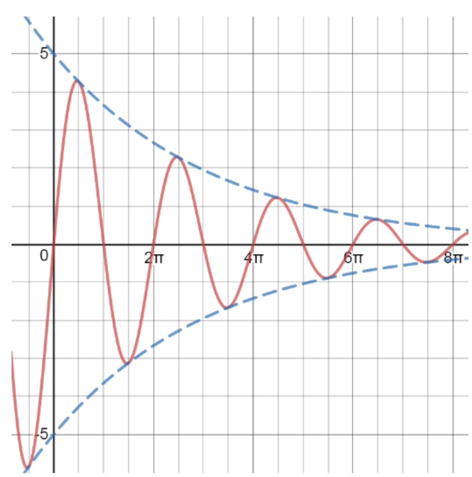
What function from your past life in algebra describes the "envelope" for the amplitude? ________________________

This behavior can be modeled using an exponential decay function as the amplitude envelope:

Here is one such decay function: $y = 5e^{-.1x}$

So our modified sine function would be written as

$f(x) = 5e^{-.1x}\sin(x)$



Carrier Waves: Another wave form can be the "envelope" for a oscillations! These is how music on the radio is "carried", by a sinusoidal "envelope" called a "carrier wave". This signal can be filtered out, then voila! You have just the actual waves that translate into music or talk or whatnot.

Variable "Midline" and Combined Waveforms: Normally D is a constant.

By hand, do a quick sketch of two periods (positive x values) of $y = \sin(x) + 2$ then dash in the Midline.

What if D is a variable?

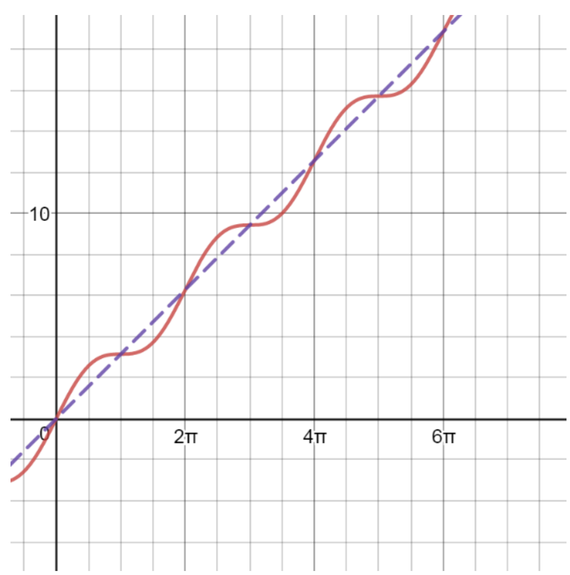
Example: $y = \sin(x) + x$

What is "D" for this function?

Predict what effect this new kind of "D" (variable) will have on the graph:

By hand, sketch y = x and predict how the sine curve will "ride" on this new "Midline"

Now graph using Desmos and see if your prediction was correct. Add the graphs of y = x in your Desmos graph.

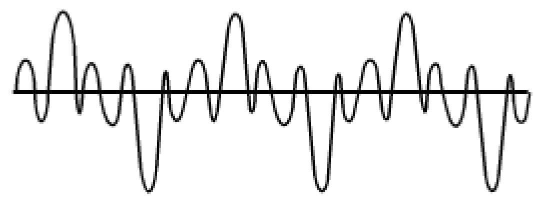


Note: The new function $y = x\sin(x)$ is NOT periodic since it doesn't return to the same values of y as it cycles through its oscillations. However, it has a distinct pattern that is important in real applications! Specifically…

Combined Wave Forms

Note the graph of the sound waves created by plucking a guitar string. Note that there is a larger oscillation with smaller oscillations “riding” the larger one.

How might we model the larger wave and the smaller “rider” waves.



Supplemental problems:

1. **Damped Oscillations.** Consider the function

$$y = 5e^{-.1x}\sin(x)$$

(a) What part of the function will control the amplitude of the sine function oscillations?

(b) Use Desmos to graph $y = 5e^{-.1x}$ from x = 0 to 6pi and y = -6 to 6. Copy this graph on your paper.

(c) Reflect the graph across the x-axis to create the “envelope”. (You can also graph $y = -5e^{-.1x}$ to see this reflection on Desmos.)

(d) Fill in sine wave oscillations on the graph within the “envelope”. The zeros are the same as for y = sin(x)

(e) Check your work by graphing $y = 5e^{-.1x}\sin(x)$ on Desmos.

(f) Did the zeros of the sine function change? Why or why not?

2. **Carrier Waves.** *Consider the function* $y = 10\cos(.1x)\sin(2x)$

(a) *Which function has the longer period (so lower frequency)* $\cos(.1x)$ or $\sin(2x)$?

(b) Graph $y = 10\cos(.1x)$ on Desmos from x = -5pi to 25pi, and sketch the graph on your paper. Next, reflect the graph across the x-axis to make an “envelope”. (You can also graph $y = -10\cos(.1x)$ on Desmos to see this reflection.)

(c) Fill in the higher frequency (shorter period) sine waves in this envelope. You don’t have to be super precise with this, just fill in a rough impression of how the sine waves would oscillate inside this envelope.

(d) Check your work by graphing $y = 10\cos(.1x)\sin(2x)$ on Desmos.

3. **Addition of functions**. Consider the function $y = \sin(x) + \frac{1}{2}x$

(a) Graph $y = \frac{1}{2}x$ by hand from x = -10 to 10

(b) Graph a sine wave “riding” on this graph.

(c) Check your work by graphing $y = \sin(x) + \frac{1}{2}x$ on Desmos.

(d) What are the x-values where the oscillations intersect the line?

4. **Combined waves:** Consider

$$y = 10\cos(.1x) + \sin(2x)$$

(a) Which wave will be the base and which will be the "rider".
(b) Use Desmos to graph the following function from x = 0to 25pi then sketch it on your paper:
$y = 10\cos(.1x)$
(c) Now add the smaller amplitude, higher frequency waves given by
$y = \sin(2x)$
(d) Check the resulting graph from (c) by graphing $y = 10\cos(.1x) + \sin(2x)$ on Desmos.
(e) How would this translate into sounds you hear?

5. Interference:

(a) Graph $y = \sin(x)$ *and* $y = \sin(x - \pi)$ by hand on the same axes.

(b) What do you expect $y = \sin(x) + \sin(x - \pi)$ *to look like?*

(c) Check your guess by graphing with Desmos.