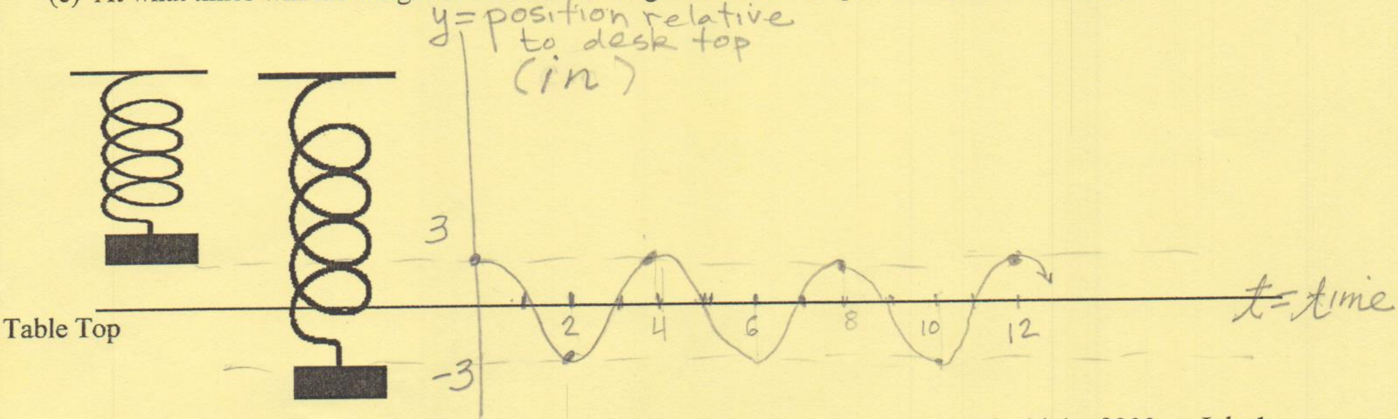
Math 229: Homework Supplement for Section 8.2 (Modeling with Sinusoidal Functions)

1. A weight attached to the end of a long spring that is bouncing up and down next to a table. As it bounces, its distance above and below the table varies sinusoidally. Assume the table top is level with the midline of the bouncing weight.

The weight is 3 inches above the table top at its highest point and 3 inches below the table top at its lowest point. Assume the weight is at its highest point at t = 0 seconds and its lowest point at t = 2 seconds.

(a) Sketch a sinusoidal graph next to the picture of the weight, showing its position relative to the table top as a function of time.
(b) Find the amplitude, period, and midline of this function.
(c) Write a cosine function to model the position of the weight relative to the table top.
(d) Where is the weight 3 seconds after it starts bouncing?
(e) At what times will the weight be at the same height at the table top? Relate this to the graph.



2. A population of animals varies sinusoidally between a low of 700 on January 1 and a high of 900 on July 1.

a. Graph the population against time.
b. Find a formula for the population as a function of time, t, measured in months since the start of the year.
c. According to your formula, what is the population on March 15?

3. Outside temperature over a day can be modeled as a sinusoidal function. Suppose you know that the high temperature for the day is 92 degrees F and the low temperature of 78 degrees occurs at 4 am. Assuming t is the number of hours since midnight, find an equation for the temperature, D, in terms of t.

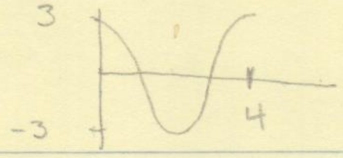
4. Sunspot Problem: For several hundred years, astronomers have kept track of the number of solar flares, or "sunspots", which occur on the surface of the sun. The number of sunspots counted in a given year varies periodically from a minimum of about 10 per year to a maximum of about 110 per year. Between the maximums that occurred in the years 1750 and 1948, there were 18 complete cycles.

a. What is the period of the sunspot cycle?

b. Assume that the number of sunspots counted in a year varies sinusoidally with the year. Sketch a graph of two sunspot cycles, starting in 1948, thus let 1948 correspond to t =0.

c. Write an equation expressing the number of sunspots per year in terms of the year

d. How many sunspots would you expect in the year 2020?

(a) 

– for reference

1. (b) Amp $= 3 \Rightarrow A = 3$

Period $= T = 4$ seconds

$$B = \frac{2\pi}{4} = \frac{\pi}{2}$$

Midline: $y = 0$

(c) $y = 3\cos\left(\frac{\pi}{2}x\right)$

(d) At $t = 3$ secs, $y = 0$ on the graph

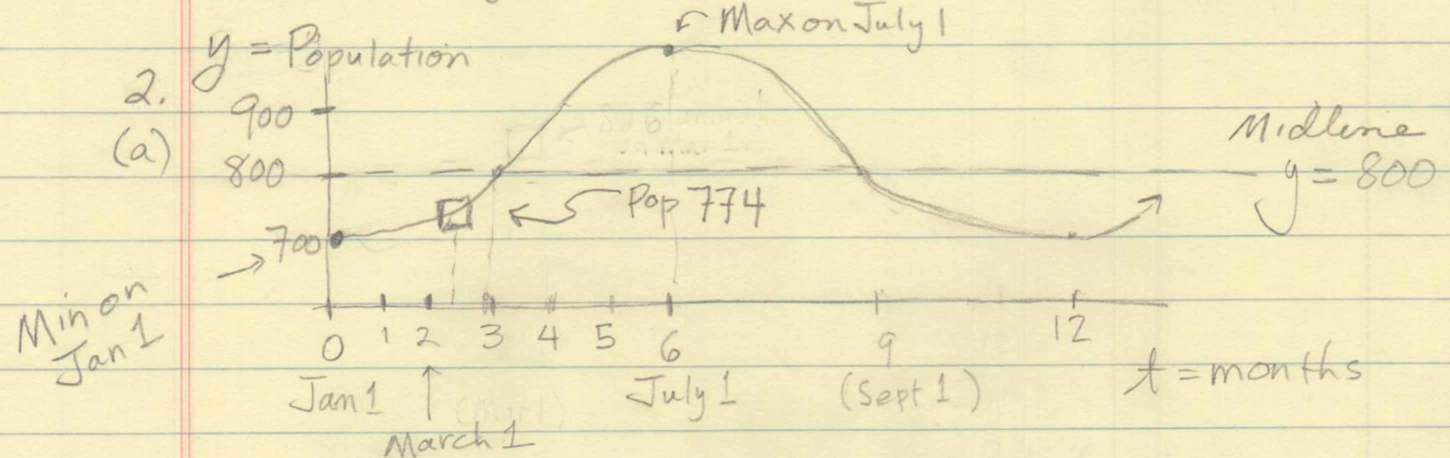
OR $y = 3\cos\left(\frac{\pi}{2}\cdot 3\right) = 3\cos\left(\frac{3\pi}{2}\right) = 3\cdot 0 = 0$

The weight is level with the desktop at $t = 3$s

(e) At $t = 1, 3, 5, 7, \ldots$ the weight is level w/desktop

$t = 2k - 1,\ k \in \mathbb{N}$ $\quad \mathbb{N}$ = the set of natural numbers $\{1, 2, 3, \ldots\}$

(Fancy answer!)

2. (a) 

(b) Amp $= 100$

$T = 12$ months

Vert shift: $+800$

$B = \frac{2\pi}{T} = \frac{2\pi}{12} = \frac{\pi}{6}$

cosine

$$y = -100\cos\left(\frac{\pi}{6}t\right) + 800$$

sine (phase shift ← 3)

$$y = -100\sin\left(\frac{\pi}{6}(t+3)\right) + 800$$

(c) March 15 $\Rightarrow t = 2.5$ $\quad y = -100\cos\left(\frac{\pi}{6}(2.5)\right) + 800 = 774$

There will be 774 animals on March 15.

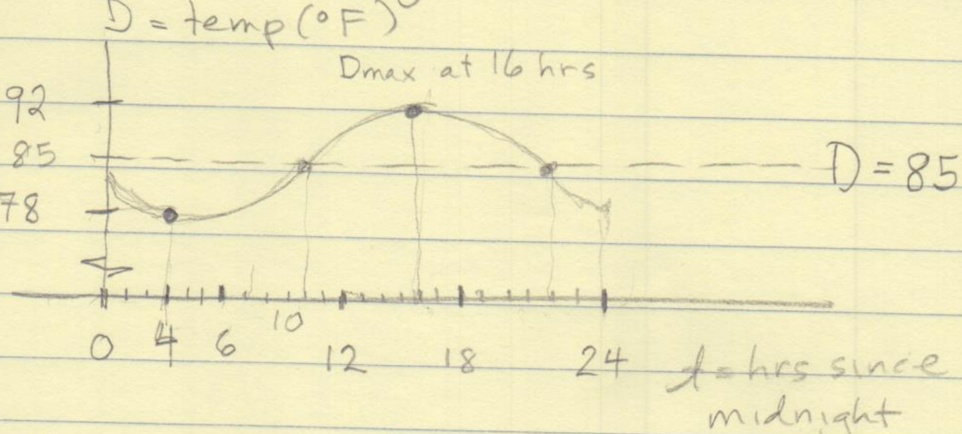
3. D = temperature

$H_{max} = 92°F$

$H_{min} = 78°F$ at 4 am

t = time since midnight (hrs)

Graph:



Amplitude:

$$Amp = \frac{H_{max} - H_{min}}{2}$$

$$= \frac{92-78}{2} = \frac{14}{2} = 7$$

$\Rightarrow$ Midline is $78 + 7 = \underline{\underline{85}}$

Period: $T = 24$ hours

Max occurs at $\frac{1}{2}$ period $= 12$ hrs

so H_{max} is at 16 hrs

$$B = \frac{2\pi}{T} = \frac{2\pi}{24} = \frac{\pi}{12}$$

Cosine: (Phase shift: $\rightarrow 4$)

$$D = -7\cos\left(\frac{\pi}{12}(t-4)\right) + 85$$

Sine: (Phase shift: $\rightarrow 10$)

$$D = 7\sin\left(\frac{\pi}{12}(t-10)\right) + 85$$

4. N = number of sunspots / year

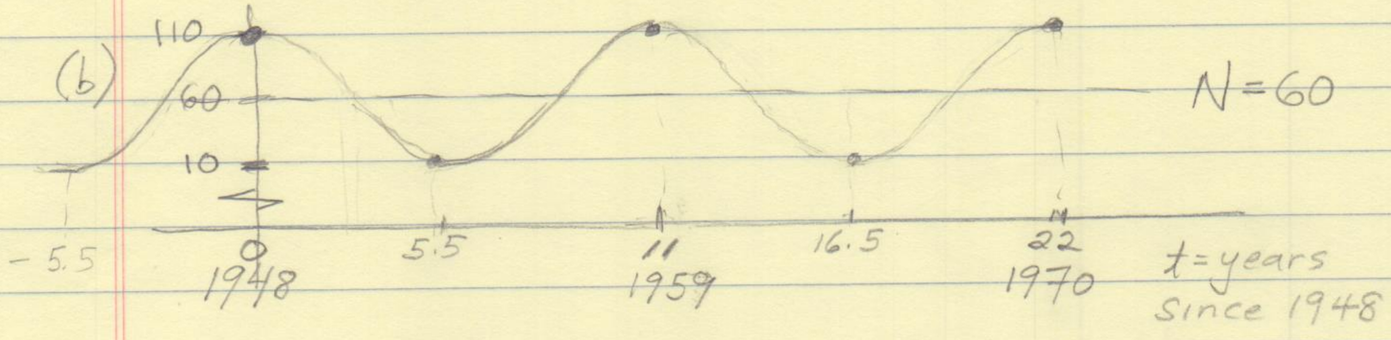
$N_{min} = 10$

$N_{max} = 110$

time for 18 cycles: $1948 - 1750 = 198$ years

(a) $T = \dfrac{198 \text{ years}}{18 \text{ cycles}} = 11$ years $\Rightarrow B = \dfrac{2\pi}{11}$

(b)



$\text{Amp} = \dfrac{110 - 10}{2} = 50$ sunspots   Midline = 60 sunspots (Average number)

cosine $\quad N = 50\cos\left(\dfrac{2\pi}{11}t\right) + 60$ (Simplest function to describe the relationship)

sine $\quad$ For sine, we could phase shift left 2.75 $\left(\frac{1}{2}(5.5)\right)$ years

$$N = 50\sin\left(\frac{2\pi}{11}(t + 2.75)\right) + 60$$