Math 229: Summary of Identities

Note 1: This is not a comprehensive list, only a list of the more commonly used identities)

Note 2: When there are general patterns that hold for all 6 trig functions, I'm using "trig(x)" to stand for any of the 6 functions.

Fundamental Identities

Reciprocal	Ratio	Even/Odd
$\sec(\theta) = \dfrac{1}{\cos(\theta)}$ (and vice versa!) $\csc(\theta) = \dfrac{1}{\sin(\theta)}$ (and vice versa!) $\cot(\theta) = \dfrac{1}{\tan(\theta)}$ (and vice versa!)	$\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)}$ $\cot(\theta) = \dfrac{\cos(\theta)}{\sin(\theta)}$	Even: $\cos(-\theta) = \cos(\theta)$ $\sec(-\theta) = \sec(\theta)$ Odd $\sin(-\theta) = -\sin(\theta)$ $\tan(-\theta) = -\tan(\theta)$ $\csc(-\theta) = -\csc(\theta)$ $\cot(-\theta) = -\cot(\theta)$

Phase shift, Cofunction, and Periodicity

Phase shift (examine the graph of each function to verify!)	Cofunction (these arise naturally from examining the complementary angles in a right triangle)	Periodicity: T = period
$\cos(\theta) = \sin(\theta + \frac{\pi}{2})$ $\sin(\theta) = \cos(\theta - \frac{\pi}{2})$	$\text{trig}(\theta) = \text{co-trig}(\frac{\pi}{2} - \theta)$ Ex: $\sin(\theta) = \cos(\frac{\pi}{2} - \theta)$ $\text{co-trig}(\theta) = \text{trig}(\frac{\pi}{2} - \theta)$ Ex: $\cos(\theta) = \sin(\frac{\pi}{2} - \theta)$	$\text{trig}(\theta + T) = \text{trig}(\theta)$ $\sin(\theta + 2\pi) = \sin(\theta)$ $\cos(\theta + 2\pi) = \cos(\theta)$ $\tan(\theta + \pi) = \tan(\theta)$ $\cot(\theta + \pi) = \cot(\theta)$ $\sec(\theta + 2\pi) = \sec(\theta)$ $\csc(\theta + 2\pi) = \csc(\theta)$

Pythagorean: Be able to derive each of these identities!

$\cos^2(\theta) + \sin^2(\theta) = 1$ $\qquad$ $1 + \tan^2(\theta) = \sec^2(\theta)$ $\qquad$ $\cot^2(x\theta) + 1 = \csc^2(\theta)$

Sum and Difference of Angles

$\cos(\alpha+\beta)=\cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta)$ $\cos(\alpha-\beta)=\cos(\alpha)\cos(\beta)+\sin(\alpha)\sin(\beta)$	$\sin(\alpha+\beta)=\sin(\alpha)\cos(\beta)+\sin(\beta)\cos(\alpha)$ $\sin(\alpha-\beta)=\sin(\alpha)\cos(\beta)-\sin(\beta)\cos(\alpha)$

$$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$\tan(\alpha-\beta) = \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

Double Angle Identities (Be able to derive cosine and sine identities from the Sum of Angles Identities)

$\cos(2\theta)=\cos^2(\theta)-\sin^2(\theta)$ $=1-2\sin^2(\theta)$ $=2\cos^2(\theta)-1$ (Be able to show how the second two can be derived from the first identity.)	$\sin(2\theta)=2\sin(\theta)\cos(\theta)$	$\tan(2\theta)=\frac{2\tan(\theta)}{1-\tan^2(\theta)}$ Be able to derive this from the cosine, sine Double Angle Identities

Reduction Formulas and Half-Angle Identities

Reduction Formulas (Be able to derive the sine/cosine identities from the Double Angle Identities for cosine.) $\sin^2(\theta)=\frac{1-2\cos(2\theta)}{2}$ $\cos^2(\theta)=\frac{1+2\cos(2\theta)}{2}$ $\tan^2(\theta)=\frac{1-2\cos(2\theta)}{1+2\cos(2\theta)}$	Half-Angle Identities: $\sin(\frac{\theta}{2})=\pm\sqrt{\frac{1-2\cos(\theta)}{2}}$ $\cos(\frac{\theta}{2})=\pm\sqrt{\frac{1+2\cos(\theta)}{2}}$ $\tan(\frac{\theta}{2})=\frac{\sin\theta}{1+\cos\theta}=\frac{1-\cos\theta}{\sin\theta}$