Math 229: Graphs of Common Polar Equations Summary

Lines in Polar Coordinates:

Lines through the Origin:

Rectangular: $y = mx$

Polar: $\theta = \theta_O$, $m = \tan(\theta_O)$

Vertical Lines

Rectangular: $x = a$

Polar: $r = a\sec(\theta)$

Horizontal Lines

Rectangular: $y = b$

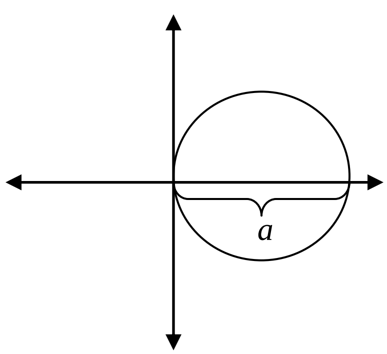
Polar: $r = a\csc(\theta)$

Circles in Polar Coordinates:

Circle Centered at the origin: $r = a$

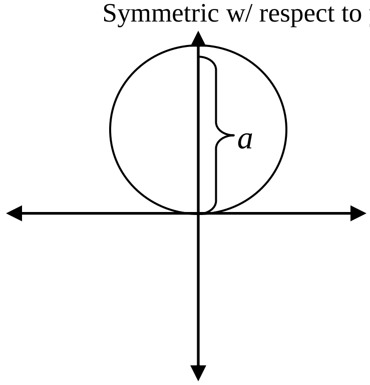
$r = a\cos\theta$

Symmetric w/ respect to **x-axis**.



$r = a\sin\theta$

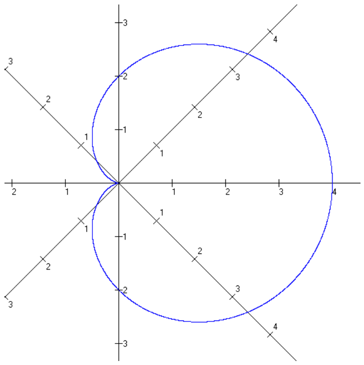
Symmetric w/ respect to **y-axis**.



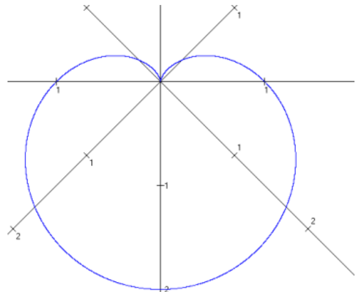
Limaçons: $r = a + b\cos\theta$ Symmetric w/ respect to **x-axis**. $r = a + b\sin\theta$ Symmetric w/ respect to **y-axis**.

To **determine shape:**

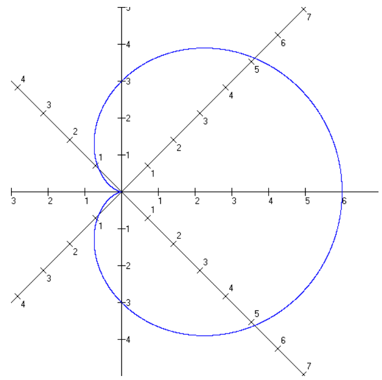
If $|a| = |b|$, creates a heart-shaped **cardiod**. These have a "cusp".



$r = 2 + 2\cos\theta$

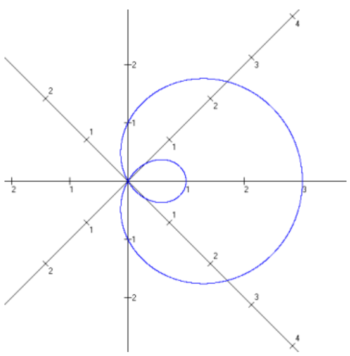


$r = -1 - \sin\theta$

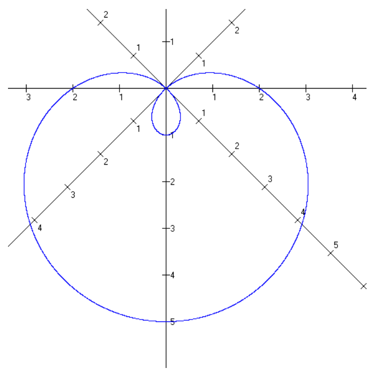


$r = -3 + 3\cos\theta$

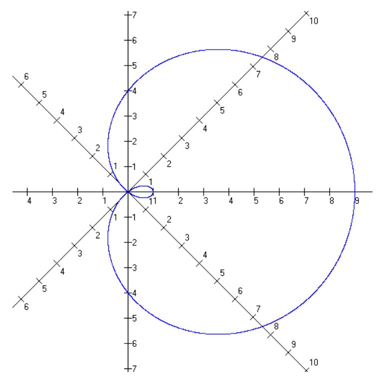
If $|a| < |b|$, creates an **inner loop**:



$r = 1 + 2\cos\theta$

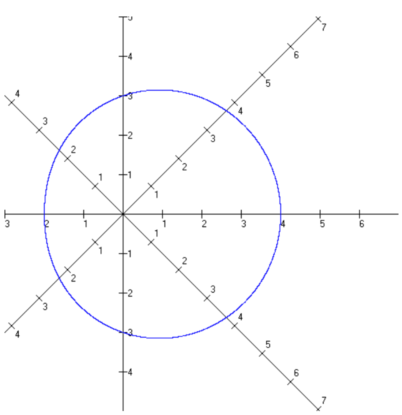


$r = 2 - 3\sin\theta$

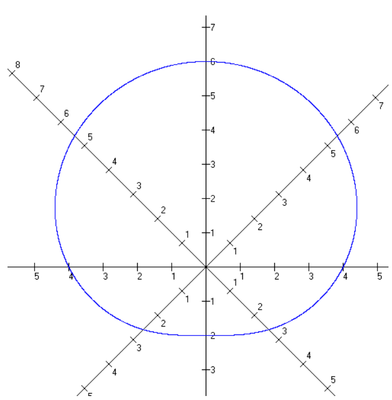


$r = -4 + 5\cos\theta$

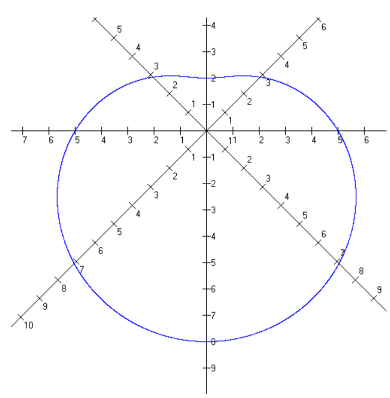
If $|a| > |b|$, creates **no** cusp **nor** inner loop. It looks like a slightly squashed circle.



$r = 3 + \cos\theta$



$r = -4 + 2\sin\theta$



$r = -5 - 3\sin\theta$

Rose Curves: ***a*** is the "height" (or length) of each petal.

$r = a\cos(n\theta)$

One petal is symmetric to x-axis,
if n is even then symmetric to both axes.

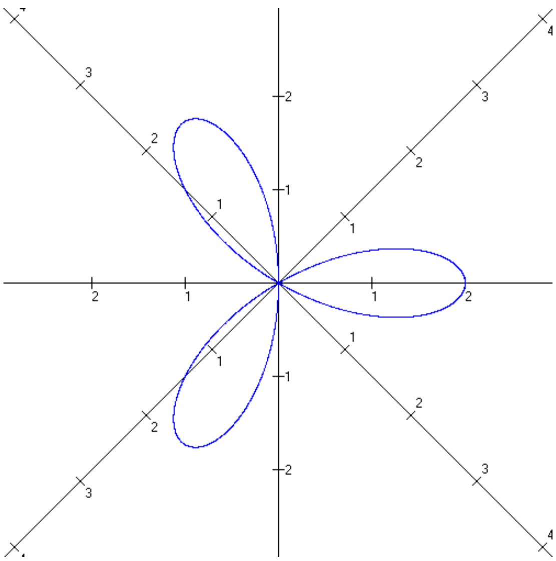
$r = a\sin(n\theta)$

May be symmetric to y-axis.

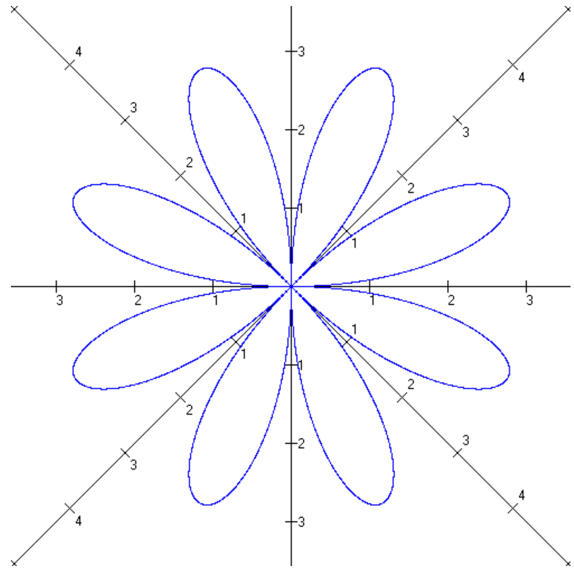
If n is odd, there will be **n petals.

If n is even, there will be **2n** petals.

Some examples:



$r = 2\cos(3\theta)$



$r = 3\sin(4\theta)$

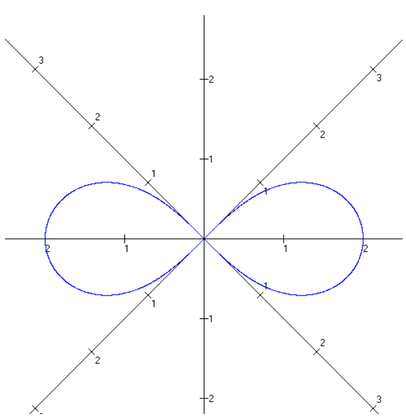
Lemniscates: ("Figure Eights")   ***a*** is the length of one loop. (One half of the figure eight.)

$r^2 = a^2\cos(2\theta)$

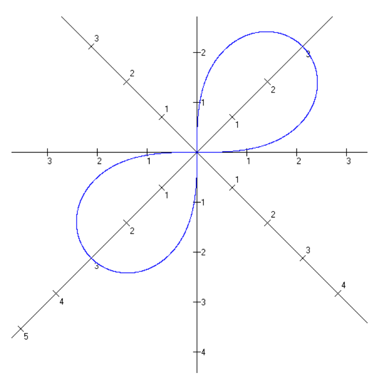
Symmetry w/ respect to x-axis, y-axis, and origin.

$r^2 = a^2\sin(2\theta)$

Symmetry w/ respect to origin only.



$r^2 = 4\cos(2\theta)$



$r^2 = 9\sin(2\theta)$