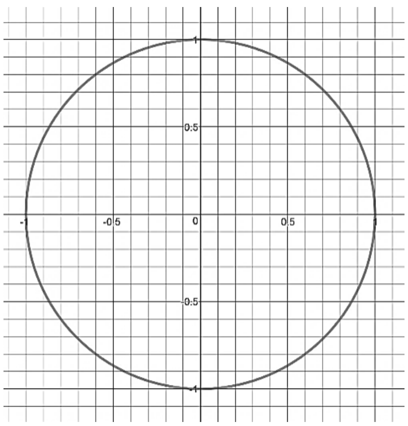
Math 229: The Unit Circle and Basic Cosine, Sine Graphs (Section 8.1)

Sinusoidal Curves

Basic (Parent) Sine Graph:

Use the unit circle to graph $y = \sin(t)$ on the (t,y) coordinate system. Make sure t is in radians!



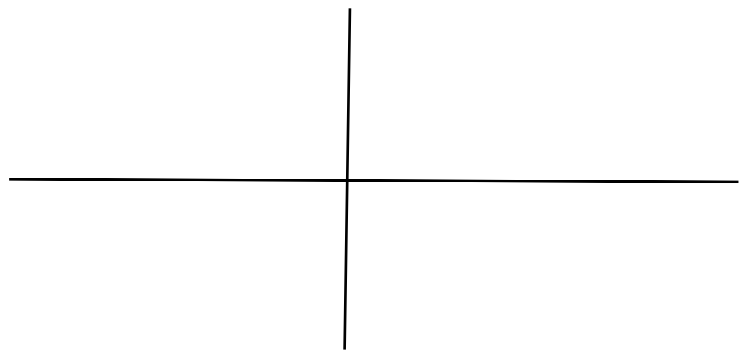


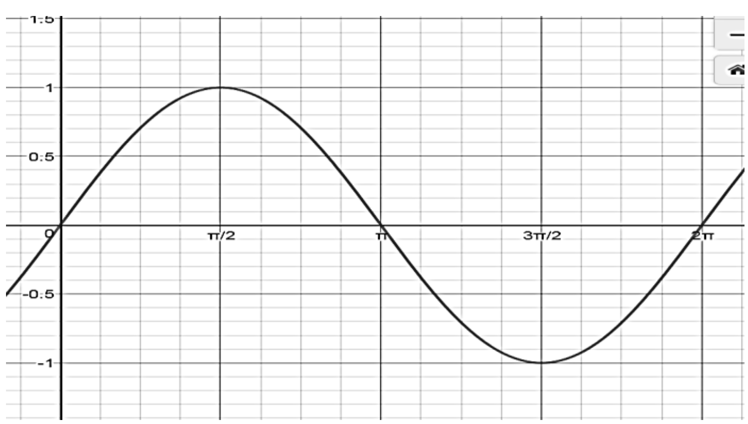
Table:

Note that $\sin(-t) = -\sin(t)$

This means sine is an ODD function: $f(-x) = -f(x)$

What type of symmetry to ODD functions have?

Basic (Parent) Graph of y = sin(x)



Features of the graph:

Midline (average value)
Amplitude

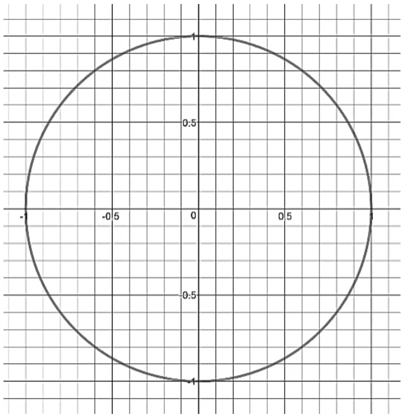
Initial (starting) Point
Period
Increment

Quarter Points

Max and Min Values
Zeros (midline points)

Basic (Parent) Cosine Graph:

Make a table using the unit circle to graph $x = \cos(t)$ on the (t,x) coordinate system. Make sure t is in radians!



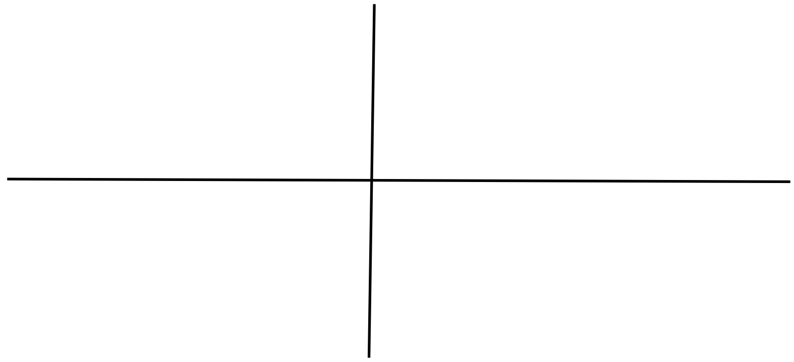
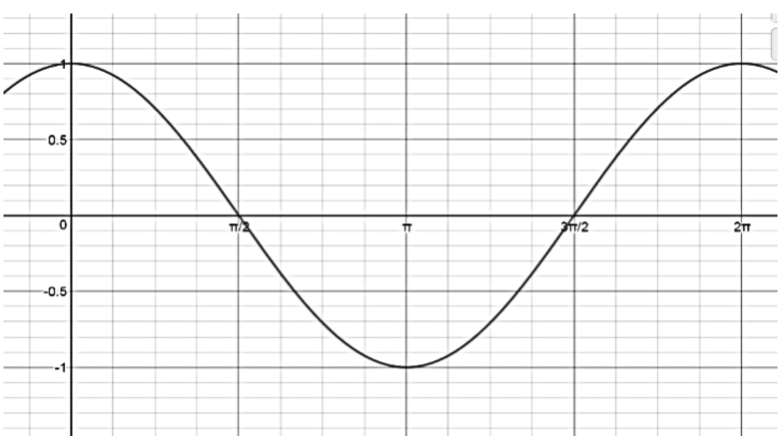


Table:

Note that $\cos(-t) = \cos(t)$

This means cosine is an EVEN function $f(-x) = f(x)$

What type of symmetry do EVEN functions have?

Basic (Parent) Graph of y = cos(x)	**Features of the graph**
	Midline (average value) Amplitude Initial (starting) Point Period Increment Quarter Points Max and Min Values Zeros (midline points)

Amplitude and Period

Amplitude and Reflection

What if the radius of the original circle was larger? What effect would that have on the sine or cosine graph?

Graph y = sin(x) and y = 3sin(x) using technology: What effect did the 3 have on the Basic Graph?

Use this observation to graph one period of $y = 2\sin(x)$ and $y = 5\cos(x)$ by hand. Check on Desmos.

Graph $y = \cos(x)$ and $y = -\cos(x)$ using technology. What effect did the negative have on the graph?

Use this observation to graph one period $y = -\sin(x)$ by hand. Check on Desmos.

Period and Quarter Points

Graph y = sin(x) and y = sin(2x) using technology. What effect did the 2 have on the Basic Graph?

Graph y = cos(x) and $y = \cos(\frac{1}{2}x)$ using technology. What effect did $\frac{1}{2}$ have on the Basic Graph?

Summary: The graph of $y = \sin(Bx)$ or $y = \cos(Bx)$ will have period, $T = \dfrac{2\pi}{B}$

If $B > 1$, then the period will be shorter (faster cycles)

If $0 < B < 1$, then the period will be longer (slower cycles)

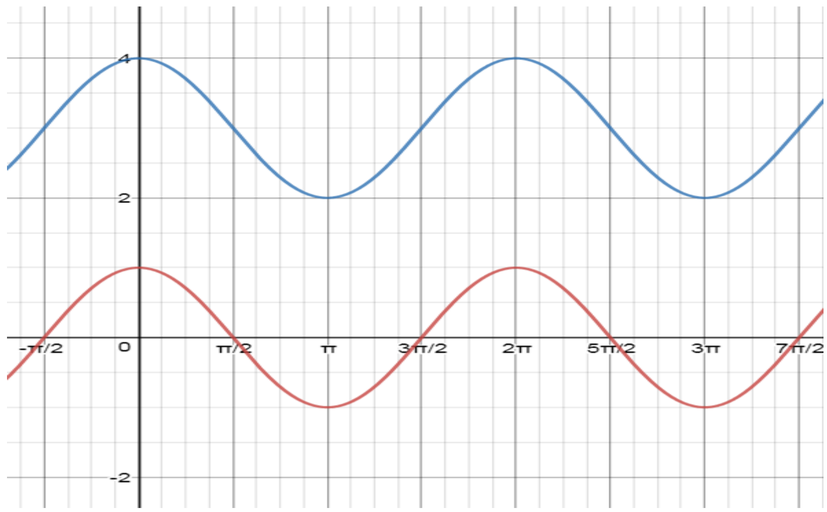
Predict: Will $y = \sin(\frac{1}{4}x)$ cycle faster or slower than $y = \sin(\frac{1}{2}x)$?

Find the period and graph each function on the same grid by hand.

Vertical Shift and Horizontal (Phase) Shift

Vertical Shift and Midline

Graph y = cos(x) and y = cos(x) + 3 using technology: What effect did the 3 have on the Basic Graph? Describe the change relative to the position of the **midline** and where the zeros went.



Horizontal (Phase) Shift

Graph $y = \sin(x)$ and $y = \sin(x - \frac{\pi}{4})$ using technology.

What effect did $\frac{\pi}{4}$ have on the graph?

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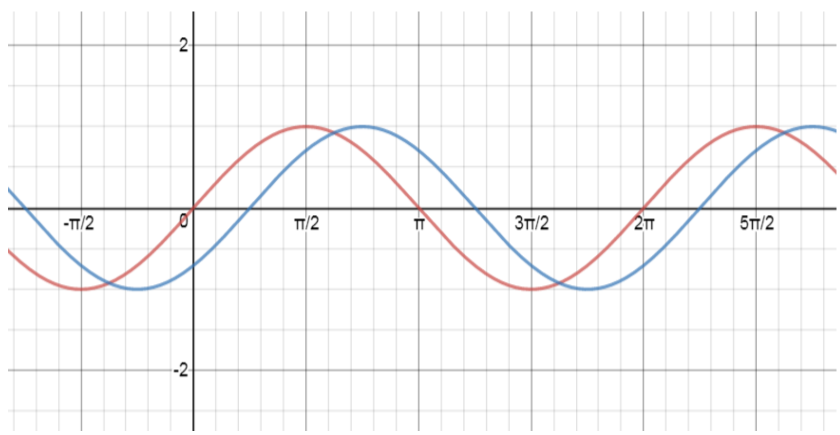

Important! In practice, modeling of periodic phenomena is almost always done using a sine function (not the cosine function).

Graph $y = \cos(x)$ and $y = \sin(x + \frac{\pi}{2})$ using technology. What do you notice

Any cosine function can be transformed into a sine function.

How? ____

Transform $y = 2\cos(3x) + 7$ into a sine function by using a Phase Shift

Answer: ____

Steps to Graph a Cosine or Sine Function $y = A\cos(B(x - x_0)) + D$ or $y = A\sin(B(x - x_0)) + D$

1. Sketch the **Basic Graph** (sine or cosine) for reference
2. **Vertical Shift**: The midline is $y = D$. Dash it in on the graph.
3. **Amplitude:** Amplitude = $|A|$. Find the amplitude of the function, and dash in the "envelope" above and below the midline.
4. **Phase Shift =** x_0 **:** The starting x-value will be x = 0 unless there has been a phase shift. If so, the phase shift, x_0, will be the starting x-value. Plot this point.
   - Sine graphs begin at the Midline
   - Cosine graphs begin at the Peak
5. **Period, Quarter Points:** $T = \frac{2\pi}{B}$ Find the period, T, then divide the period by 4 to get the increment. Use the increment to plot the quarter points from the starting x-value.
   Fill in the midline points, max, and min values now.
   Extend the pattern for the second period.
6. **Reflection:** If A < 0 (negative), then there is a vertical reflection. Reflect the max and min across the midline if there is a reflection.

Connect the points in a smooth, sinusoidal curve!

Example: Graph two periods of each of the following by hand, then check your work using Desmos:

$y = 4\sin(\tfrac{1}{2}x)$

$y = 3\sin(x) + 5$

$y = \cos(\pi(x-1))$

$y = -2\cos(x)$

$y = \cos(\frac{\pi}{3}x) + 4$