#3 problem 2, #17 West not east! Be methodical!

Math 229: Test 1 **Name:** KEY

(100 points)

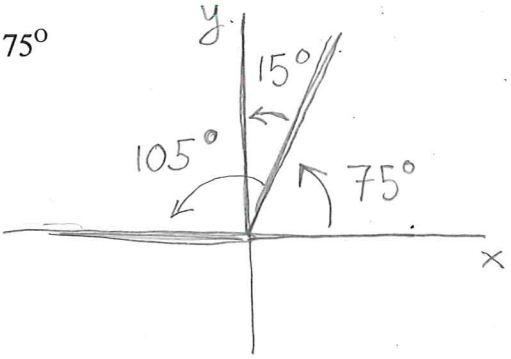
Please do your work in a well-organized manner. Credit is based on the amount of correct work shown, not just on the final answer. Give exact answers and use proper notation. Only *scientific calculators* are allowed on the exam.

1. (2 pts) Which number is considered "EXACT" and which is an approximation?

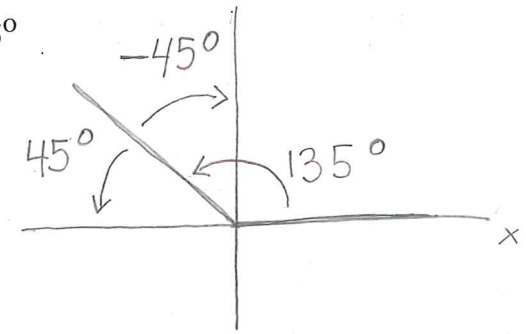
   $\sqrt{2}$ is EXACT and 1.414 is an approximation of $\sqrt{2}$

2. (4 pts) Sketch each of the angles in standard position on the xy-coordinate system:

   a. 75°

   b. 135°

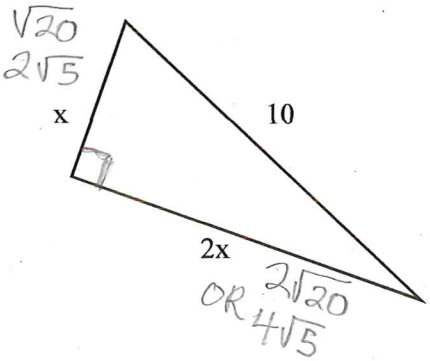
   

3. (6 pts) Find the complement and supplement of the angles in Problem 2 and sketch each on the graph.

   a. Complement: 15° Supplement: 105°

   b. Complement: -45° Supplement: 45°

4. (6 pts) Solve for x in the right triangle. Leave your answer in exact terms.

   OR $\sqrt{20}$ OR $2\sqrt{5}$

   OR $2\sqrt{20}$ OR $4\sqrt{5}$

   $x^2 + (2x)^2 = 10^2$

   $x^2 + 4x^2 = 100$

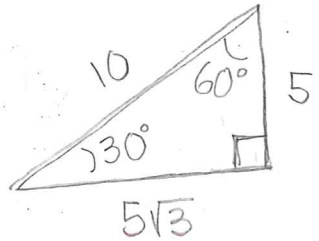
   $5x^2 = 100$

   $x^2 = 20$

   $x = \pm\sqrt{20}$

   side length: $x = \sqrt{20}$ OR $2\sqrt{5}$

5. (5 pts) What are the other sides of a 30-60-90 triangle if the shortest side is 5? Give the answers as exact values (leave square roots as square roots). Sketch a picture of the triangle with the sides labeled with their corresponding values.

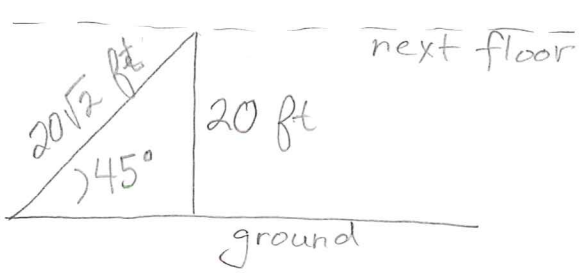
   

   short leg = 5

   long leg = $5\sqrt{3}$

   hypotenuse = 10

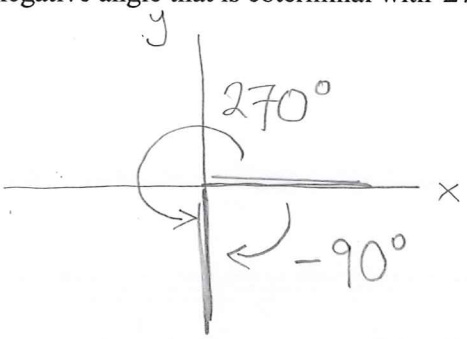
6. (5 pts) An escalator is to carry people a vertical distance of 20 feet between floors. How long is the escalator if it makes an angle of 45° with the ground? Include a sketch in your solution.



Escalator is $20\sqrt{2}$ feet length

Approx: 28.3 feet

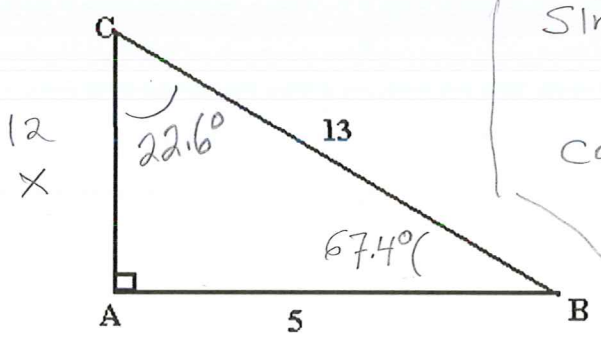
7. (5 pts) Find a negative angle that is coterminal with 270°. Illustrate both angles on a single graph.



Coterminal angle: -90°

8. (10 points) (a) Find the values of all six trig functions for angle B using the given triangle.

7



$\sin B = \frac{12}{13}$ $\quad \csc B = \frac{13}{12}$

$\cos B = \frac{5}{13}$ $\quad \sec B = \frac{13}{5}$

$\tan B = \frac{12}{5}$ $\quad \cot B = \frac{5}{12}$

$x^2 + 5^2 = 13^2$

$x^2 + 25 = 169$

$x^2 = 144$

$x = \pm\sqrt{144} = 12$ (pos only)

3 (b) Find the angles in the triangle above. Show work and clearly label which angle is which.

(Many possible set ups for solution.)

$\cos B = \frac{5}{13}$

$B = \cos^{-1}\left(\frac{5}{13}\right) = 67.4^\circ$

$B = 67.4^\circ$

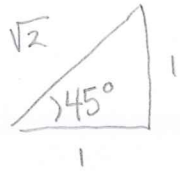
$C = 90^\circ - 67.4^\circ$

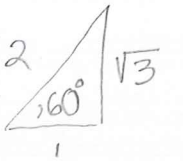
$C = 22.6^\circ$

9. (5 pts) Explain why it is impossible for $\sin\theta$ to be equal to any number greater than 1.

$\sin\theta = \frac{\text{opposite leg}}{\text{hypotenuse}}$. The hypotenuse is always the longest side in a right triangle so $\frac{\text{opposite}}{\text{hypotenuse}}$ will never be greater than 1.

10. (5 pts) By hand, evaluate the expression $\left(\sin^2(45^\circ)+\cos(60^\circ)\right)^2$. Use exact values NOT approximations. Simplify your answer as much as possible.





$$\left[(\sin 45^\circ)^2 + \cos 60^\circ\right]^2$$

$$= \left[\left(\frac{1}{\sqrt{2}}\right)^2 + \frac{1}{2}\right]^2$$

$$= \left[\frac{1}{2} + \frac{1}{2}\right]^2 = 1^2 = \boxed{1}$$

Draw a triangle for each angle if in doubt!

11. (5 pts) By hand, convert the angle $40^\circ 54'$ to decimal degrees.

$40^\circ 54' = 40^\circ + \frac{54}{60}^\circ = \boxed{40.9^\circ}$

12. (8 pts) Use your calculator to evaluate each of the following. Write your answer to 3 decimal places of accuracy.

a. $\tan(54^\circ) =$ 1.376

b. $\sin(112^\circ) =$ .927

b. $\csc(4.8^\circ) =$ 11.951

$= \frac{1}{\sin(4.8^\circ)}$

d. $\cot(89^\circ) =$ .017

$\frac{1}{\tan(89^\circ)}$

13. (5 pts) Use your calculator to find the angle in each of the equations below:

a. $\sin\theta = .7660$

$\theta = \sin^{-1}(.7660)$

$\theta = 49.996^\circ$

Better: $\theta = 50.00^\circ$

4 sig figs

b. $\sec\theta = 1.923$

$\frac{1}{\cos\theta} = \frac{1}{1.923}$

$\cos\theta = .5200$

technically, we shouldn't round this number

$\theta = \cos^{-1}(.5200)$

$\theta = 58.67^\circ$

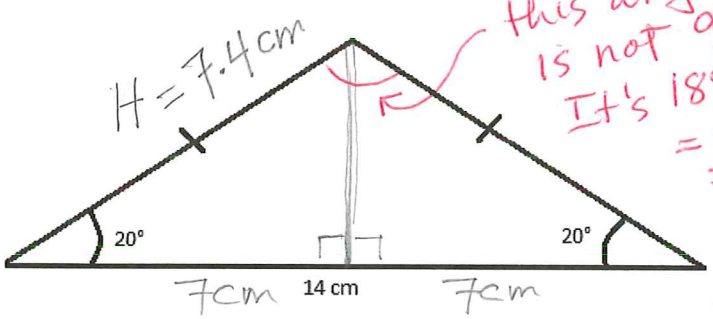
4 sig figs

14. (5 pts) If the base of an isoceles triangle is 14 cm and each of the two equal angles measures 20°, find the length of the two equal sides.

$$\cos 20^\circ = \frac{7}{H}$$

$$H = \frac{7}{\cos 20^\circ}$$

$$H = 7.4 \text{ cm}$$



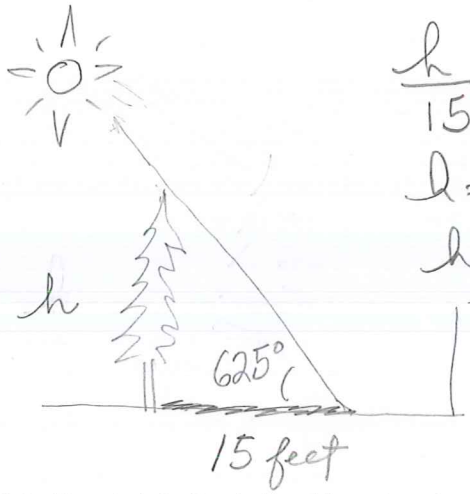
this angle is not 90° It's 180°-40° = 140°

15. (5 pts) If the angle of elevation of the sun is 62.5°, how tall is a tree that casts a shadow that is 15 feet long?

For full credit, include a sketch in your solution.

switch angle -2

Discuss



$$\frac{h}{15} = \tan 62.5^\circ$$

$$h = 15 \tan 62.5^\circ$$

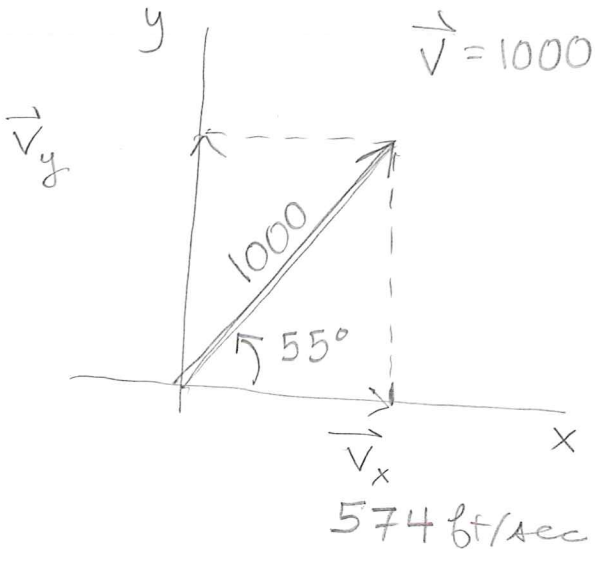
$$h = 28.8 \text{ feet}$$

The tree is 28.8 (or 29) feet tall.

16. (5 pts) A bullet is fired into the air with an initial velocity of 1000 feet per second at an angle of 55°. How fast the bullet is traveling horizontally and vertically? (i.e., Find the x and y-vector components of the velocity).

Discuss notation v_x vs. $\vec{v}_x$ vs. $|\vec{v}_x|$

For full credit, include a sketch in your solution.



$\vec{v} = 1000 \frac{\text{ft}}{\text{sec}}$ at 55°

$$\frac{\vec{V}_y}{1000} = \sin 55^\circ$$

$$\vec{V}_y = 1000 \sin 55^\circ$$

$$\vec{V}_y = 819 \text{ ft/sec}$$

$$\frac{\vec{V}_x}{1000} = \cos 55^\circ$$

$$\vec{V}_x = 1000 \cos 55^\circ$$

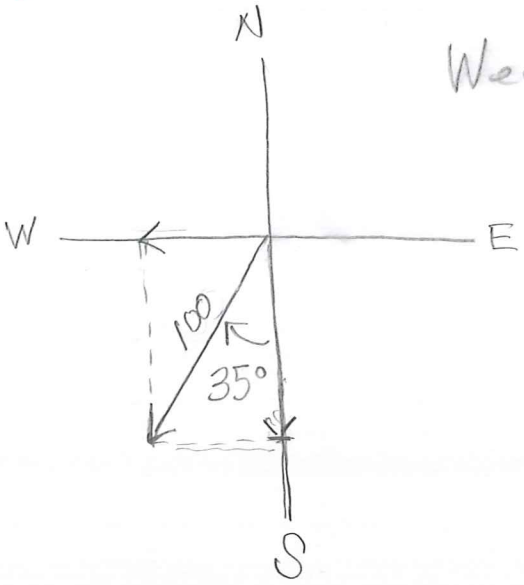
$$\vec{V}_x = 574 \text{ ft/sec}$$

Discuss internal rounding

West!

17. (10 pts) (a) A ship traveled 100 miles on a bearing of $S\ 35^{\circ}W$. How far south and how far ~~east~~ has the ship gone?

6 For full credit, include a sketch in your solution.



West distance $= 100 \sin 35^{\circ}$

West = 57.4 miles

57 mi better

South distance $= 100 \cos 35^{\circ}$

South = 81.9 miles

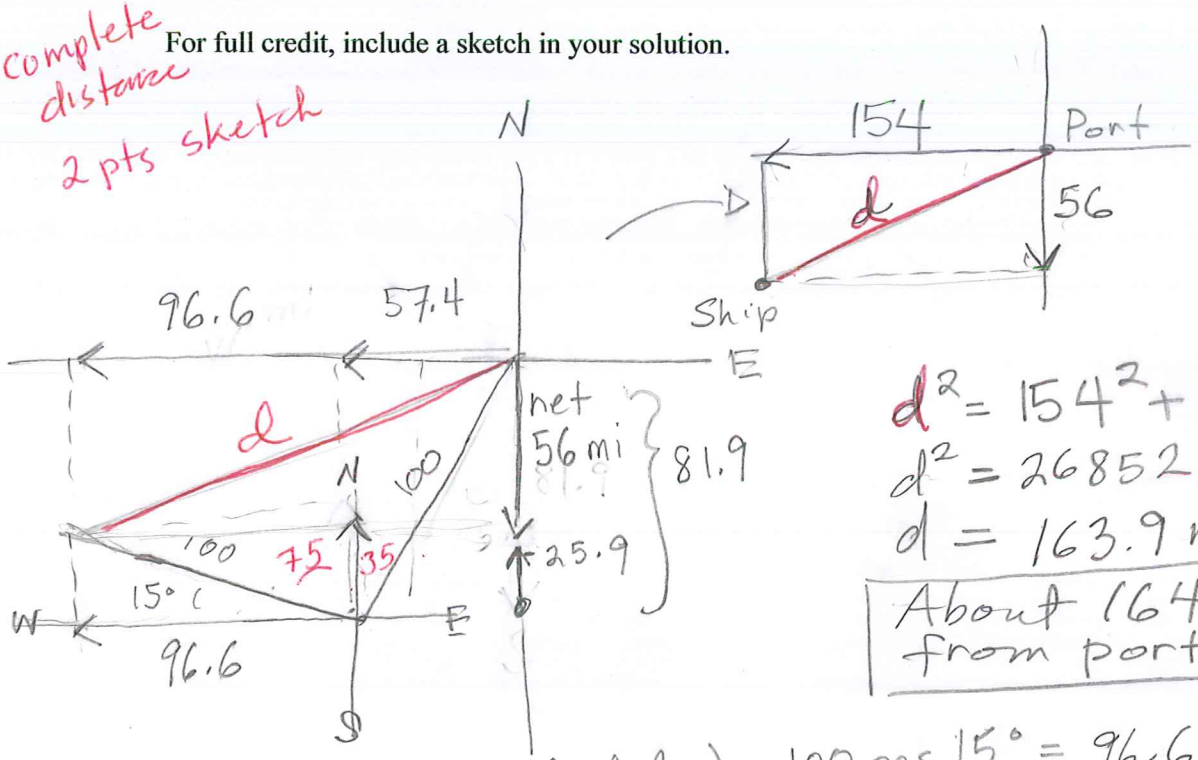
82 mi better

+1 +1 XC (b) If the ship changes its bearing to $N\ 75^{\circ}W$ and travels another 100 miles, how far will the ship be from the port at which is began its journey?

complete distance

For full credit, include a sketch in your solution.

2 pts sketch



$d^2 = 154^2 + 56^2$

$d^2 = 26852$

$d = 163.9$ mi

About 164 miles from port.

West distance (2nd leg) $= 100 \cos 15^{\circ} = 96.6$ mi

Total west $= 96.6$ mi $+ 57.4$ mi $= 154$ mi

North distance (2nd leg) $= 100 \sin 15^{\circ} = 25.9$ mi

South − North $= 81.9$ mi $- 25.9$ mi $= 56$ mi