Math 229: Test 1 (100 points)
(Wright, Fall 2018)

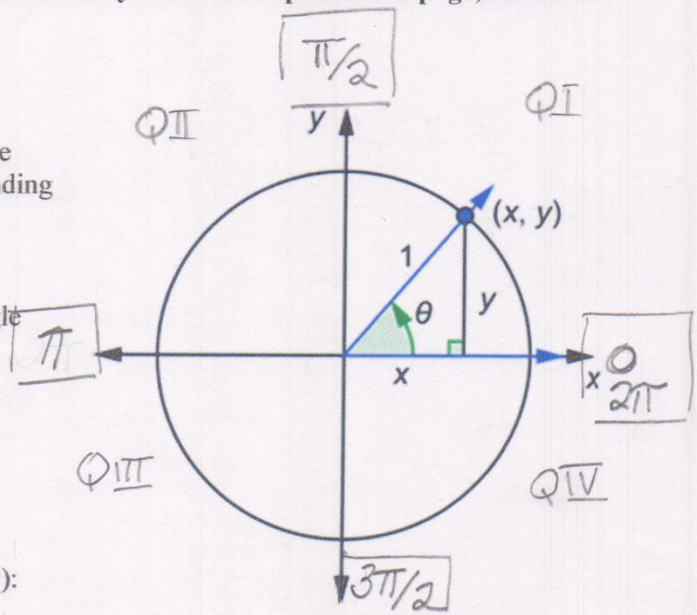
Name:_____

In Class: _____/80 points **Take Home:** _____/20 points

Part 1: This portion of the exam will be done with no calculator. Once you have completed this page, turn it in then take out your calculator for Part 2.

Box your answers.

1. (4 pts) (a) In the picture of the Unit Circle shown, label the positive and negative x-axis and y-axis with the corresponding radian angle.



(b) If (x, y) is a point on the Unit Circle, and $\theta =$ the angle created by (x, y), as shown, then

x = $\cos\theta$

y = $\sin\theta$

2. (5 pts) Evaluate each of the following (no work necessary):

$\sin(0) = 0$ $\cos(0) = 1$ $\sin\left(\frac{\pi}{2}\right) = 1$ $\cos\left(\frac{3\pi}{2}\right) = 0$ $\sin(2\pi) = 0$

3. (3 pts) Evaluate: $\sin\left(\frac{3\pi}{2}\right)\cos(\pi)$.

$= (-1)(-1)$

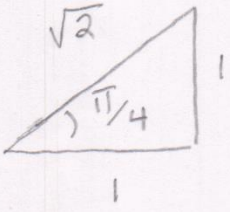
$= 1$

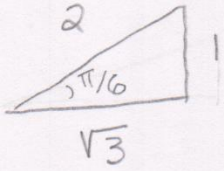
note: $\cos\pi = -1$

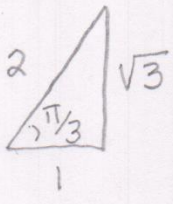
$\sin\left(\frac{3\pi}{2}\right) = -1$

4. (6 pts) From memory or by sketching the appropriate triangle, evaluate the following:

(a) $\tan\left(\frac{\pi}{4}\right) = 1$ (b) $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ (c) $\sec\left(\frac{\pi}{3}\right) = 2$







$\pi/4 = 45°$ $\frac{\pi}{6} = 30°$ $\frac{\pi}{3} = 60°$

When you've finished, turn in this part of the test and pick up Part 2.

Test 1, Part 2: Calculator Portion **Name:**_____

Part 2: Calculator allowed (but not necessary for most problems!):

For full credit, you must show correct work.

1. (3 pts) Convert the angle $\frac{7\pi}{5}$ to degrees (no sketch needed!):
   Express your answer to the nearest tenth of a degree.
   rounding not necessary!

$\frac{7\pi}{5} \cdot \frac{180^\circ}{\pi} = 252^\circ$

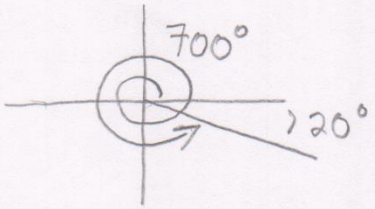
$= 252.0^\circ$

2. (3 pts) Convert the angle 220° to radians (no sketch needed!):
   Express your answer as a reduced fraction.

$220^\circ \cdot \frac{\pi}{180^\circ} = \frac{11\pi}{9}$

3. (5 pts) Sketch 700° in standard position and find a positive and a negative coterminal angle.

Sketch:



$700^\circ - 360^\circ = 340^\circ$

Positive coterminal angle: 340°

Negative coterminal angle: -20°

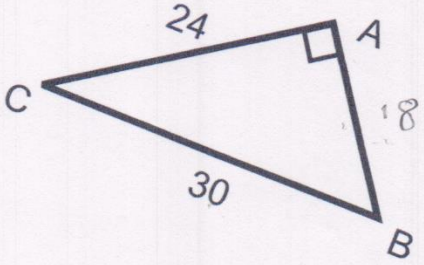
4. (10 points) Evaluate the trig functions using the given triangle.
   Express the answers as reduced fractions.

$a^2 + b^2 = c^2$

$24^2 + b^2 = 30^2$

$b^2 = 324$

$b = 18$



$\sin(B) = \frac{24}{30} = \frac{4}{5}$

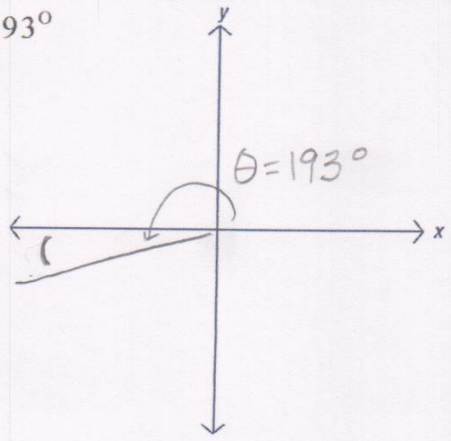
$\tan(B) = \frac{24}{18} = \frac{4}{3}$

$\cos(C) = \frac{24}{30} = \frac{4}{5}$

$\csc(C) = \frac{30}{18} = \frac{5}{3}$

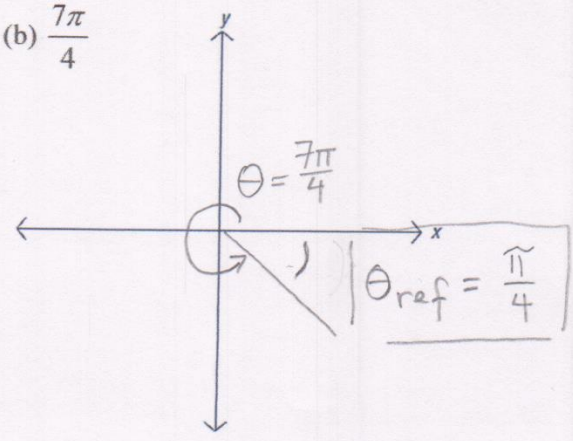
5. (4 pts) Sketch the angle and label the reference angle.

(a) 193°



$\theta_{Ref} = 13^\circ$

(b) $\frac{7\pi}{4}$



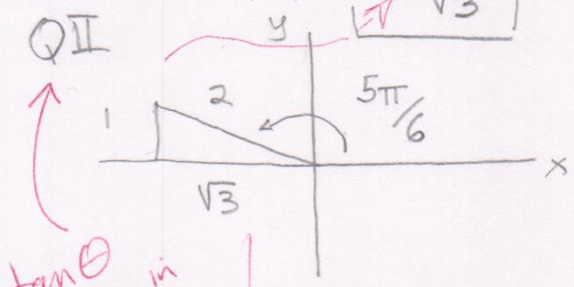
$\theta_{ref} = \frac{\pi}{4}$

− +

6. (4 pts) If sin(t) < 0, and tan(t) > 0, which quadrant is the terminal side of angle t in? QIII

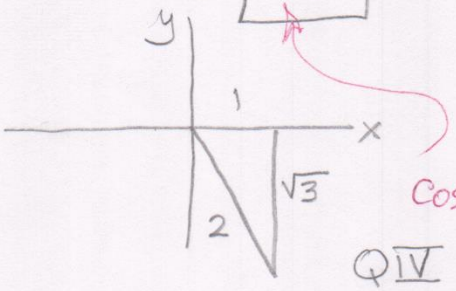
7. (10 pts) Find the exact value of each of the following, using reference angles.

(a) $\tan\left(\frac{5\pi}{6}\right) = -\frac{1}{\sqrt{3}}$



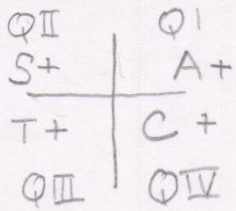
tanθ is neg. in QII | 2 points

(b) $\cos\left(\frac{5\pi}{3}\right) = +\frac{1}{2}$



QIV

Cosθ is pos in QIV



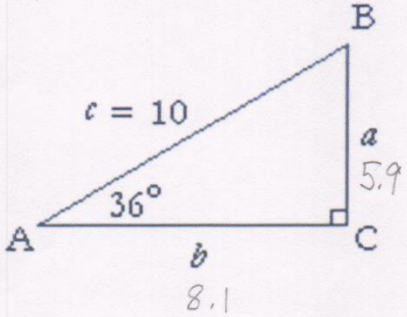
8. (8 pts) Evaluate the following using your calculator: Rounded to 4 decimal points (answers will vary)

$\sin\left(\frac{7\pi}{12}\right) = .9659$ $\csc(137^\circ) = \frac{1}{\sin(137^\circ)} = 1.4663$

$\sec(-12^\circ) = 1.0223$ $\cot\left(\frac{\pi}{5}\right) = 1.3764$

$= \frac{1}{\cos(-12^\circ)}$ $= \frac{1}{\tan(\pi/5)}$

9. (10 pts) Find the missing sides of the given triangle:



$\frac{a}{10} = \sin 36^\circ$

$a = 10\sin 36^\circ$

$a = 5.8779$

$\frac{b}{10} = \cos 36^\circ$

$b = 10\cos 36^\circ$

$= 8.0902$

OR $a^2 + b^2 = c^2$

$5.8779 + b^2 = 10^2$

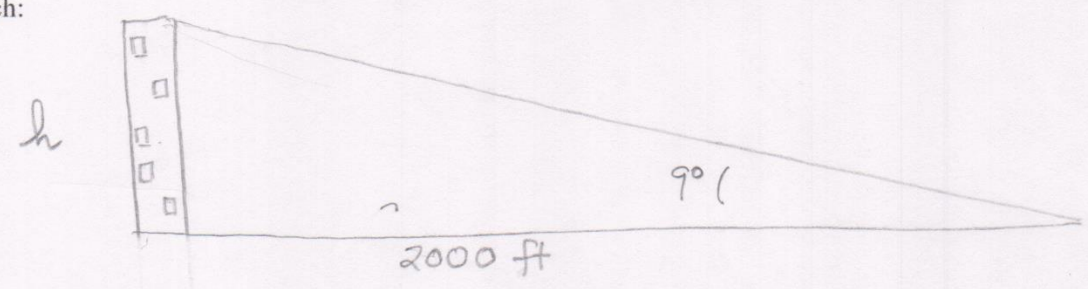
$b^2 = 65.4503$

$b = 8.0901$

10. (6 pts) The angle of elevation to the top of a building in Chicago is found to be 9 degrees from the ground at a distance of 2000 feet from the base of the building. Using this information, find the height of the building.

Hard to draw to scale! ☺

Sketch:



$\frac{h}{2000'} = \tan 9^\circ \Rightarrow h = 2000'\tan 9^\circ \Rightarrow h = 316.8'$

The building is $316.8 \approx 317$ feet tall

Math 229: Test 1 Take Home **Name:** _____
(20 points)

This exam is due at the beginning of class on Wednesday, 9/12/18. You may work with other people in the class but not with tutors, other instructors, etc. .

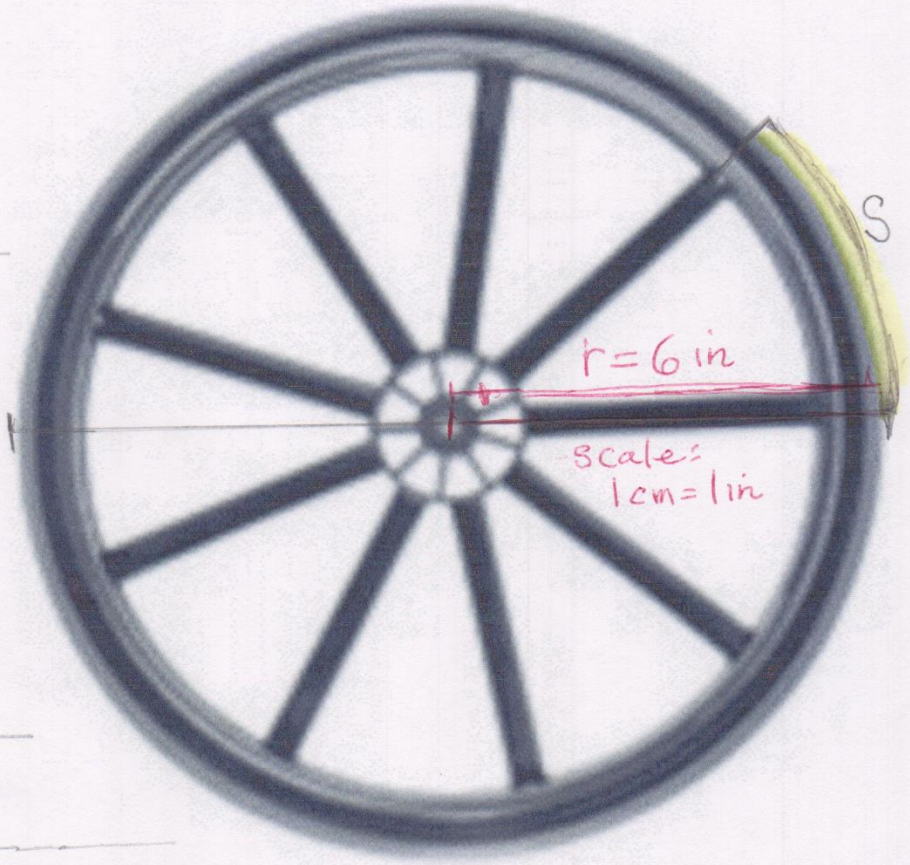
- *Your work should be clear and well-organized. Messy or scribbled out work will lose 2 points overall on the exam. (You may do your work on another sheet of paper if you find there isn't enough room on this exam.)*
- *Scoring will be based on the completeness and accuracy of your work, not just the final answer.*

1. (9 pts) Suppose that a nine-spoke wheel has a diameter of 12 inches. Find the length of the piece of the wheel that lies between two spokes using the method specified below. *Round your answer to two decimal places (1/100th of an inch).*

4 (a) Use the arclength formula. Show work and write the result on the picture of the wheel in an appropriate spot.

$S = r\theta = 6\text{in}\left(\frac{2\pi}{9}\right) = \frac{4\pi}{3}\text{ in}$

$\theta = \frac{2\pi}{9}$ ($\frac{1}{9}^{th}$ of a full rev.)



Calculated distance: $S = 4.19$ in

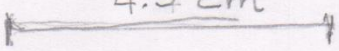
3 (b) Use entire circumference of the wheel then determine what proportion of the circumference is between spokes. Show work.

$C = 2\pi r = 2\pi(6\text{in}) = 12\pi\text{ in}$

$S = \frac{1}{9}C = \frac{12\pi}{9}\text{ in} = \frac{4\pi}{3}\text{ in}$

Calculated distance: $S = 4.19$ in

2 (c) The image of the wheel is set so that the diameter is 12 cm on this paper (use a ruler to check this), which means 1 cm on the picture corresponds to 1 inch on the actual wheel. Use a piece of string then the ruler to directly measure the distance on the wheel from one spoke to another. my string length 4.3 cm



Measured distance: $S = 4.3\text{cm} = 4.3\text{in}$

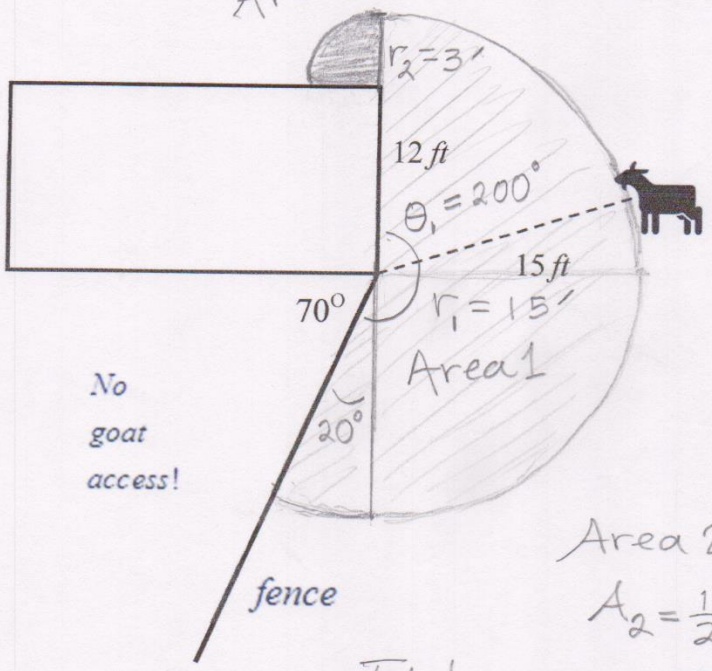
Did your measured distance exactly match the calculated distance? No

What does this tell you about using two decimal places of accuracy in your original calculations (parts (a) and (b))?

Without a really precise way of measuring, we can't achieve a precise answer. This is why the sciences emphasize significant figures. Our answers should reflect the 2 sig figures in the original problem, hence 4.2 in would be better in (a) & (b).

2. (6 pts) Consider the picture of the goat tied to the side of a barn. The side of the barn is 12 feet long as shown, and there is a fence on one side (as shown below) that the goat can't get through. Determine how much area the goat has for grazing. (Hint: Think in terms of circle areas and sector area!)

For full credit, illustrate your reasoning on the diagram below.



Area 1: $\theta_1 = 180° + 20° = 200°$

In radians: $\theta_1 = 200° \cdot \frac{\pi}{180°} = \frac{10\pi}{9}$

$r_1 = 15$ ft

$A_1 = \frac{1}{2}\theta_1 r_1^2 = \frac{1}{2}\left(\frac{10\pi}{9}\right)(15\text{ ft})^2 = \frac{1125\pi}{9}\text{ ft}^2 = 392.7\text{ ft}^2$

Area 2: $\theta_2 = \frac{\pi}{2}$ $r_2 = 3$ ft

$A_2 = \frac{1}{2}\theta_2 r_2^2 = \frac{1}{2}\cdot\frac{\pi}{2}\cdot(3\text{ ft})^2 = \frac{9\pi}{4}\text{ ft}^2 = 7.1\text{ ft}^2$

Total area $= 392.7 + 7.1 = \boxed{399.8\text{ ft}^2}$

3. (4 pts) Linear and Angular Speed

The best hitters in Major League Baseball have an average bat swing speed of about 70 mph at the moment of impact with the ball. Assuming the distance from the shoulder to the "sweet spot" (center of mass of the bat and best location to hit the ball) is 48 inches, what is the angular speed of a player at the moment he hits the ball?

Show your work for all unit conversions necessary for this problem!

Linear speed

$V = 70\frac{\text{mi}}{\text{hr}} \cdot \frac{5280\text{ ft}}{1\text{ mi}} \cdot \frac{1\text{ hr}}{3600\text{ sec}} = 102.\overline{6}\frac{\text{ft}}{\text{sec}}$

$r = 48\text{ in} = 4\text{ ft}$

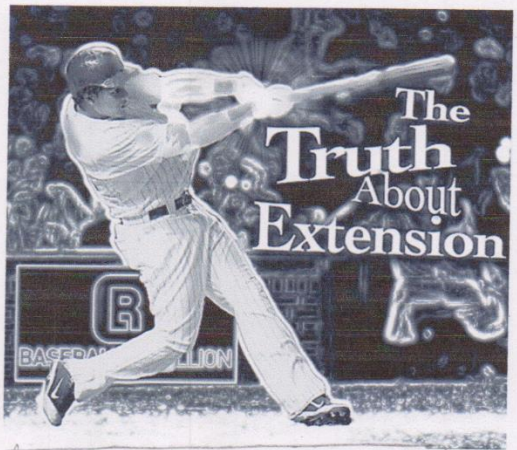
$\omega = ?$ (units: $\frac{\text{radians}}{\text{sec}}$ – unitless!)

$V = r\omega$

$70\frac{\text{mi}}{\text{hr}} = 48\text{ in}\cdot\omega$ ← unlike units! See conversions above!

$\frac{102.\overline{6}\frac{\text{ft}}{\text{sec}}}{4\text{ ft}} = \frac{4\text{ ft}\cdot\omega}{4\text{ ft}}$

$\omega = 102.\overline{6}\frac{\text{ft}}{\text{sec}}\cdot\frac{1}{4\text{ ft}} = 25.\overline{6}\frac{\text{radians}}{\text{sec}}$



$\omega \approx 26\frac{\text{radians}}{\text{sec}}$