• Discuss notation (argument)

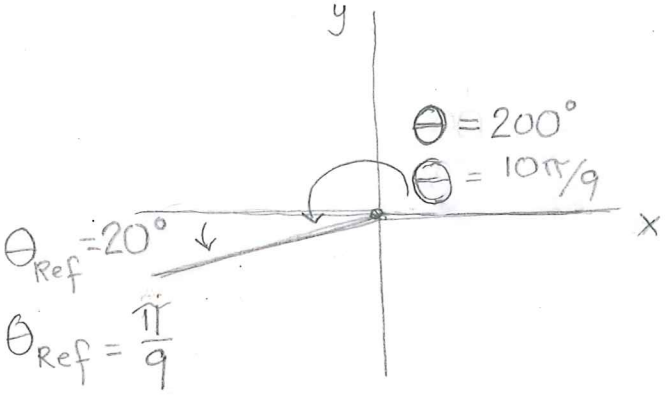
Math 229: Test 2 **Name:** KEY

(100 points)

Please do your work in a well-organized manner. Credit is based on the amount of correct work shown, not just on the final answer. Give exact answers and use proper notation. Only *scientific calculators* are allowed on the exam.

1. (8 pts) (a) Sketch the angle 200° in standard position on the xy-coordinate system and give the value of the reference angle. Label the reference angle on the graph.

4



$\theta_{REF} = 20^\circ$

$\theta_{REF} = \frac{\pi}{9}$

4 (b) Convert both the angle and the reference angle to radian measure (in exact values). Put them on the graph above.

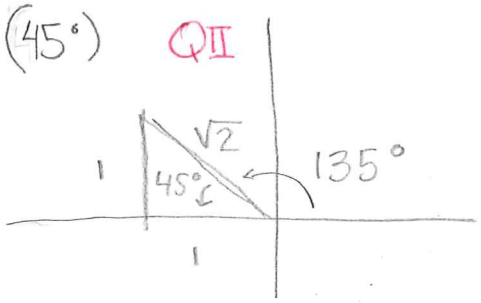
$\theta = 200^\circ \cdot \frac{\pi}{180^\circ} = \frac{10\pi}{9}$

$\theta_{REF} = 20^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{9}$

2. (8 pts) Use reference angles to find the exact value of each of the following. Show your work in sketching the angle, the reference angle, and the corresponding triangle.

4 (a) $\cos(135^\circ) = -\cos(45^\circ)$ QII

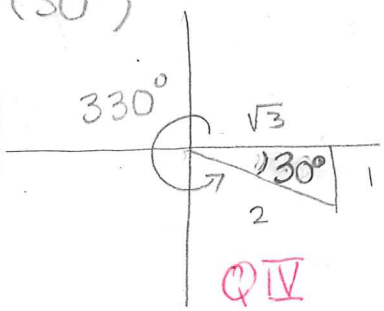
$= -\frac{1}{\sqrt{2}}$



Cosine is negative in QII!

4 (b) $\cot(330^\circ) = -\cot(30^\circ)$

$= -\frac{\sqrt{3}}{1} = -\sqrt{3}$



QIV

Cotangent is negative in QIV

Watch notation!

Write

"$\cot 330^\circ = -\sqrt{3}$"

NOT

"$\cot = -\sqrt{3}$"

1½ sign

1 angle, triangle

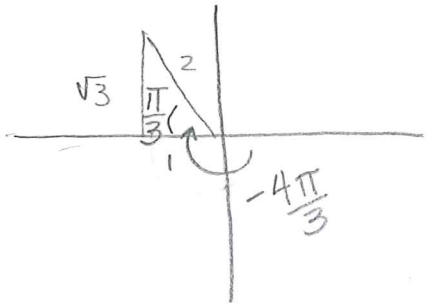
1½ value

3. (8 pts) Use reference angles to find the exact value of each of the following. Show your work in sketching the angle, the reference angle, and the corresponding triangle.

(c) $\sin\left(-\frac{4\pi}{3}\right)$

$= \sin\frac{\pi}{3}$

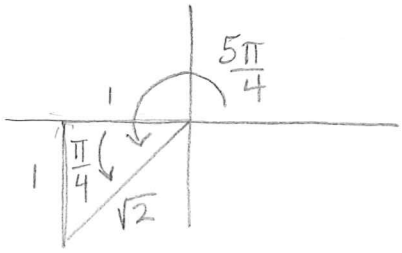
$= \frac{\sqrt{3}}{2}$



(d) $\sec\left(\frac{5\pi}{4}\right)$

$= -\sec\left(\frac{\pi}{4}\right)$

$= -\frac{\sqrt{2}}{1} = -\sqrt{2}$



$\theta_{RAD} = \frac{s}{r} = \frac{1}{1}$, so $s = r$

4. (8 pts) Use the given unit circle to roughly estimate the angle given in radians (show the angle on the graph) and the values of the trig functions.

ANSWERS WILL VARY

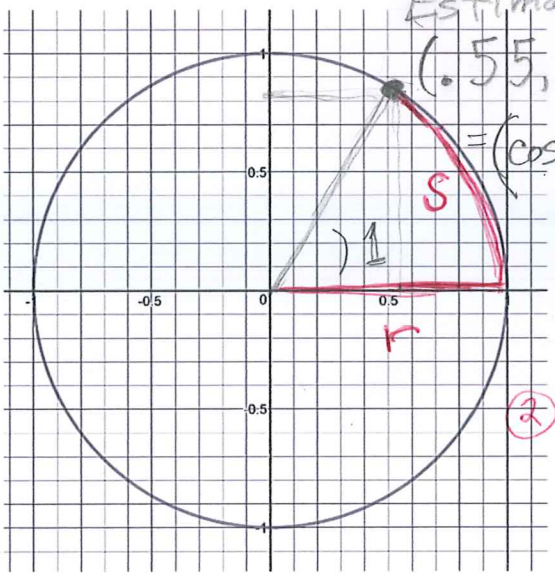
1 (a) $\cos(1) = .55$

1 (b) $\sin(1) = .85$

2 (c) $\tan(1) = \frac{.85}{.55} = 1.54$

Note: The argument of "1" is in radians

1 radian is the angle subtended by an arc of length 1.



Estimate (2)

(.55, .85)

$= (\cos(1), \sin(1))$

(2)

5. (4 pts) Evaluate each of the following: ALL are ZERO ✓

$\sin(0) = 0$ $\sin(\pi) = 0$ $\sin(2\pi) = 0$ $\sin(3\pi) = 0$ $\sin(-\pi) = 0$

$\cos\left(\frac{\pi}{2}\right) = 0$ $\cos\left(\frac{3\pi}{2}\right) = 0$ $\cos\left(\frac{5\pi}{2}\right) = 0$ $\cos\left(\frac{7\pi}{2}\right) = 0$ $\cos\left(-\frac{\pi}{2}\right) = 0$

6. (6 pts) Use your calculator to estimate the following values (to 4 decimal places).

(a) $\cos(15.2^\circ)$

$= .9650$

(b) $\cot(234^\circ) = \frac{1}{\tan(234^\circ)}$

$= .7265$

(c) $\tan\left(\frac{5\pi}{9}\right)$

$= -5.6713$

round up

(d) $\csc(3.4) = \frac{1}{\sin(3.4)}$

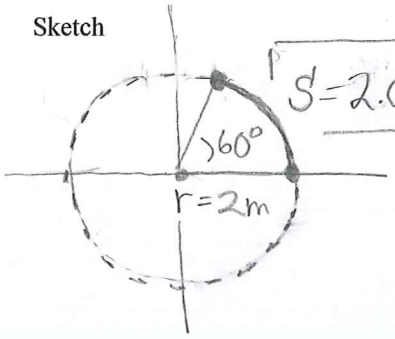
$= -3.9133$

round up!

7. (6 pts) If $\theta = 60^\circ$ is the central angle in a circle of radius 2 meters, find the length of the arc cut off by θ. Give the value in exact terms and as an approximation to two decimal places.

For full credit, include a sketch of the circle and the arc described.

Sketch



$s = 2.09\ m$

$60^\circ = \frac{\pi}{3}$ radians $= \theta_{RAD}$

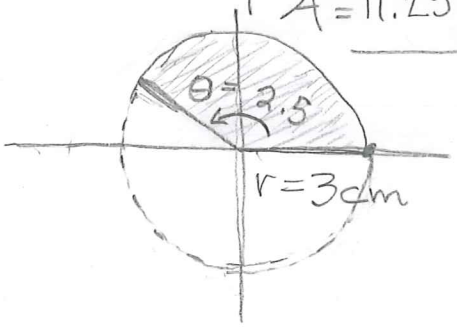
$s = r\theta_{RAD} = 2m\left(\frac{\pi}{3}\right) = \frac{2\pi}{3}\ m$

$= 2.09\ m$

8. (6 pts) Find the area of a sector formed by central angle $\theta = 2.5$, with radius of the circle 3 centimeters.

For full credit, include a sketch of the circle and the sector described.

Sketch



$A = 11.25\ cm^2$

$\theta_{RAD} = 2.5$

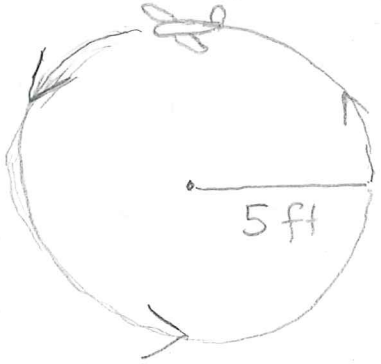
$A = \frac{1}{2}\theta_{RAD} r^2$

$= \frac{1}{2}(2.5)(3cm)^2$

$= 11.25\ cm^2$

XC

9. (5 pts) A girl is spinning a model airplane on a string that is 5 feet long. If she twirls the plane at 1.5 revolutions per minute, how far does the plane travel in 20 seconds?



$\theta_{RATE} = 1.5\ \frac{rev}{min} \times \frac{2\pi\ rad}{rev} = 3\pi\ \frac{radians}{min}$ } angular velocity

$s_{RATE} = r\theta_{RATE} = 5\ ft\left(3\pi\ \frac{rads}{min}\right) = 15\pi\ \frac{feet}{min}$

$d = RATE \cdot TIME$

$= 15\pi\ \frac{feet}{min} \times 20\ Seconds \times \frac{1\ min}{60\ sec}$

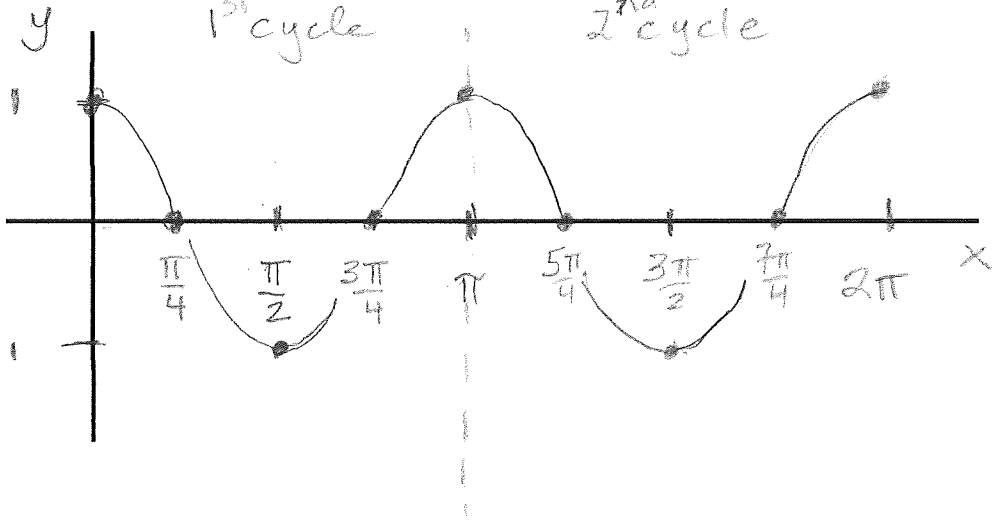
$= 5\pi\ ft$

$= 15.7\ ft$

10. (6 points) Graph $y = \cos(2x)$ from $x = 0$ to $x = 2\pi$. Clearly label the axes with key values.

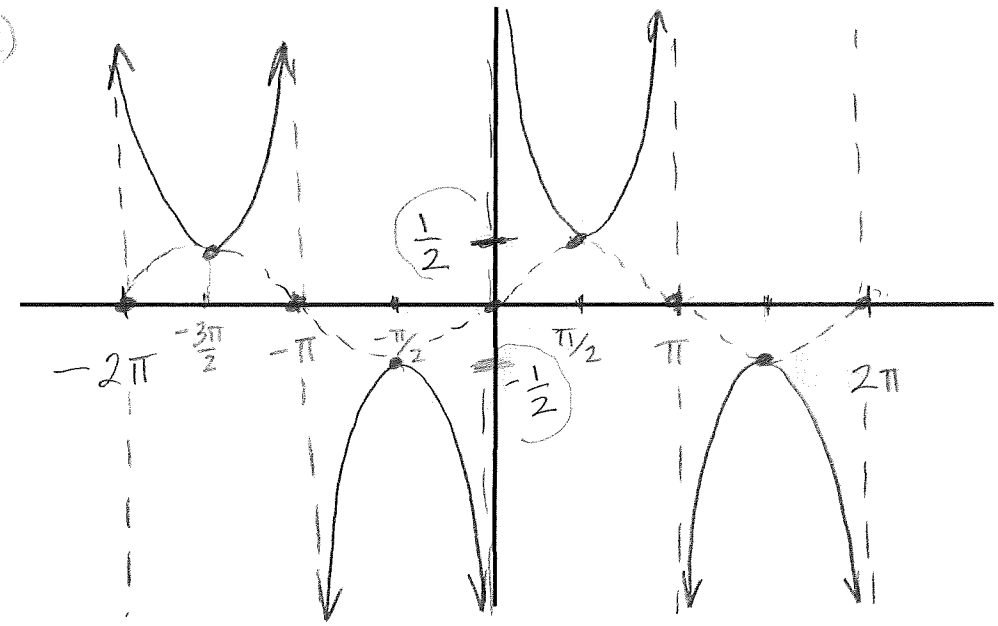
$B = 2$

$T = \frac{2\pi}{B} = \frac{2\pi}{2} = \pi$



11. (6 points) Graph $y = \frac{1}{2}\csc(x)$ from $x = -2\pi$ to $x = 2\pi$. Clearly label the axes with key values.

Graph $y = \frac{1}{2}\sin(x)$
Fill in V.A.'s
Fill cosecant



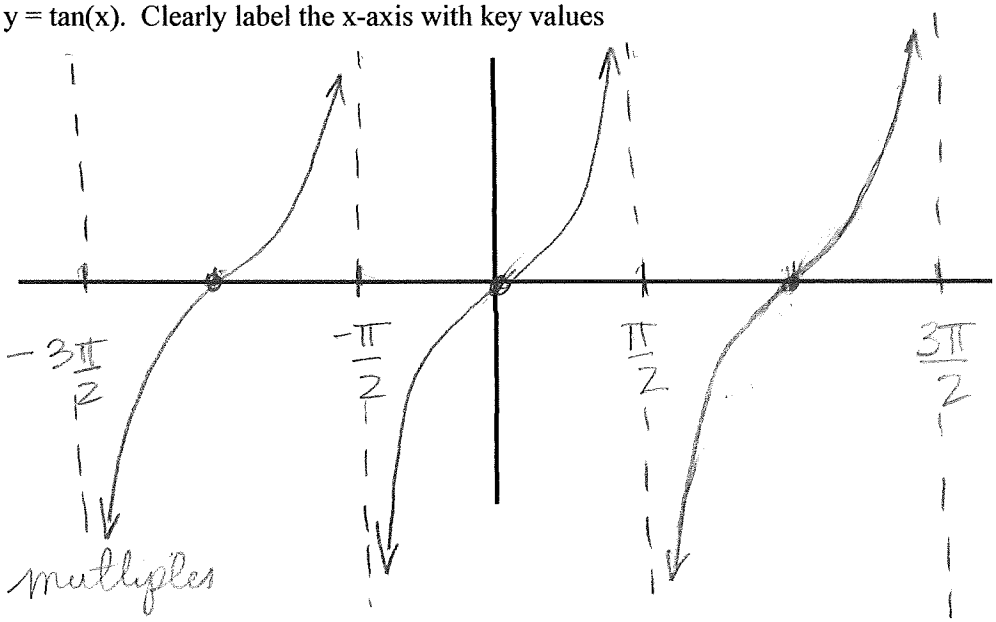
12. (6 points) Graph 3 periods of y = tan(x). Clearly label the x-axis with key values

V.A's where there is division by zero

$y = \tan(x)$

$= \frac{\sin(x)}{\cos(x)}$

zero at odd multiples of $\pi/2$



13. (10 points) Given $y = -3\sin(\pi x) + 2$, identify each of the following, then graph two complete cycles (periods) of the function.

vertical reflect, ~~by~~ 5

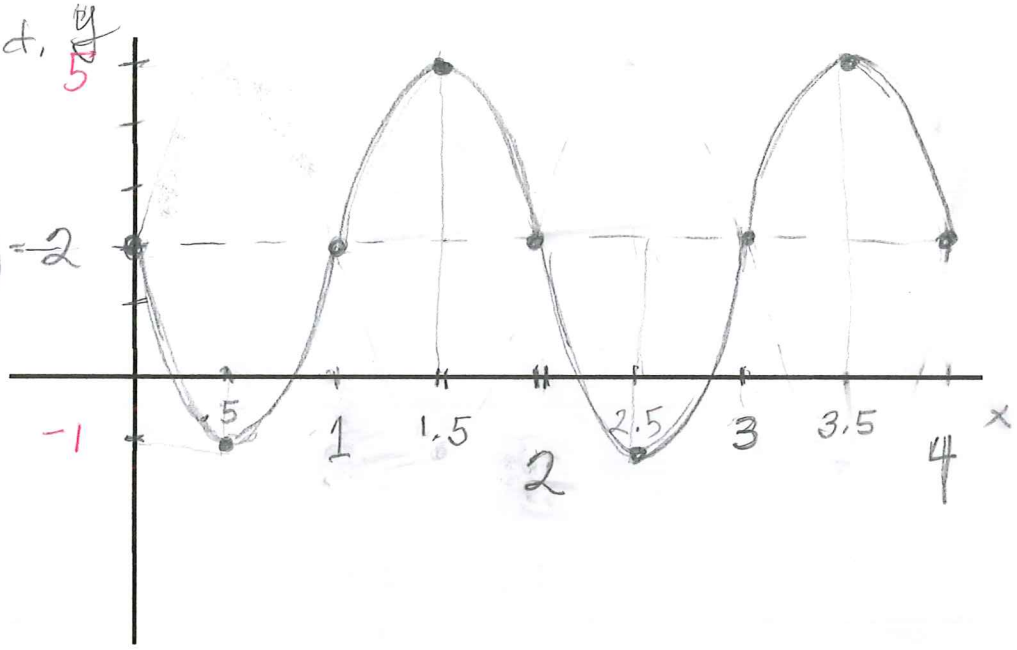
Amplitude: 3

Period: 2

Vertical shift: 2

$B = \pi$

$T = \frac{2\pi}{\pi} = 2$



14. (5 pts) Determine the phase (horizontal) shift of the given function: $y = \cos(2x - \frac{\pi}{2})$. You do not have to graph!

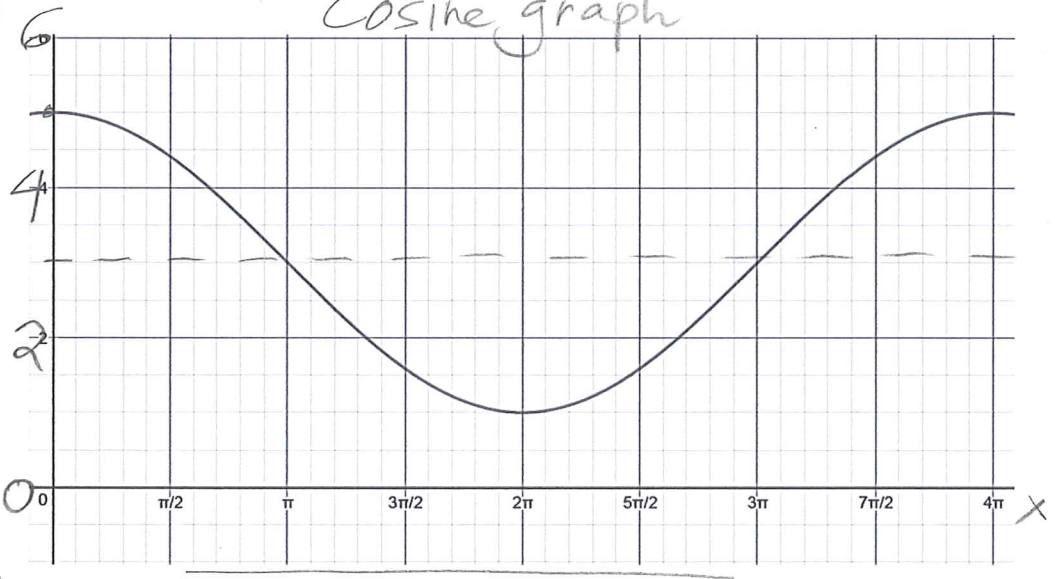
$2x - \frac{\pi}{2} = 0$

$2x = \frac{\pi}{2}$

$x = \frac{\pi}{4}$ Phase shift is right $\frac{\pi}{4} \approx .79$ units

15. (8 pts) Determine the equation of the function for the graph shown below:

Cosine graph



$T = 4\pi$

$B = \frac{2\pi}{T} = \frac{2\pi}{4\pi} = \frac{1}{2}$

Max = 5, Min = 1

Midline = Avg = $\frac{1+5}{2}$

$y = 3$ (vertical shift)

phase shift: none

$A = 2$ (by inspection)

$A = \frac{\text{Max} - \text{Min}}{2} = \frac{5-1}{2} = 2$

$y = 2\cos(\frac{1}{2}x) + 3$