Math 229: Test 2 (Wright, Fall 201~~9~~8) **Name:** KEY
(100 points)

Please do your work in a well-organized manner. Credit is based on the amount of correct work shown, not just on the final answer. Give exact answers where asked for and use proper notation. Only scientific calculators are allowed on the exam.

1. (16 points) Sketch two full periods (one on the right and one on the left of the y-axis) of each of the following functions. For full credit, the x-axis and y-axis must be labeled with all relevant values (quarter points, midline, max/min values)

9 points marked on each graph.

8 (a) $y = 2\sin x + 3$

Amp $B=1$ (so usual period of 2π) Midline

Amplitude: 2

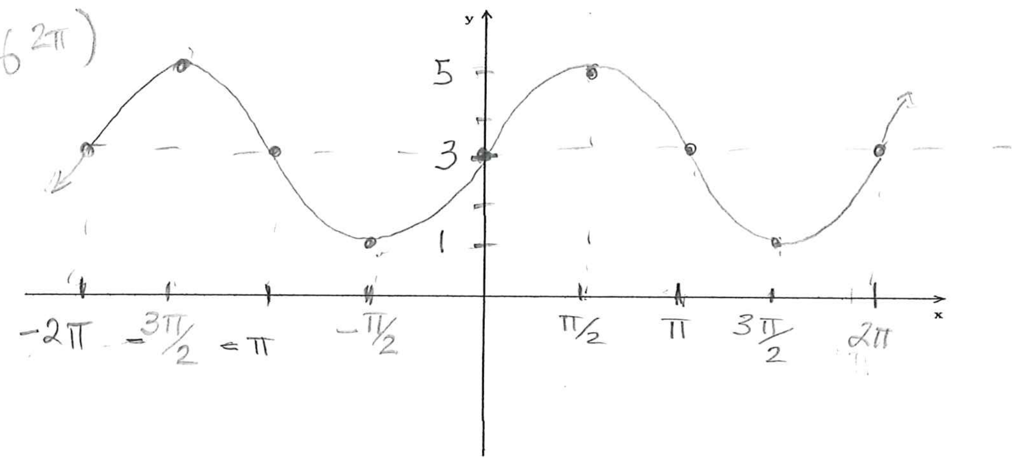
Period: $T = 2\pi$

Midline: $y = 3$

Domain: $(-\infty, \infty)$

Range: $[1, 5]$

some interpreted as restricted domain - ok



8 (b) $y = -3\cos\left(\frac{\pi}{3}x\right)$

reflect across x-axis amp $B = \frac{\pi}{3}$

$$T = \frac{2\pi}{\frac{\pi}{3}} = 2\pi \cdot \frac{3}{\pi} = 6$$

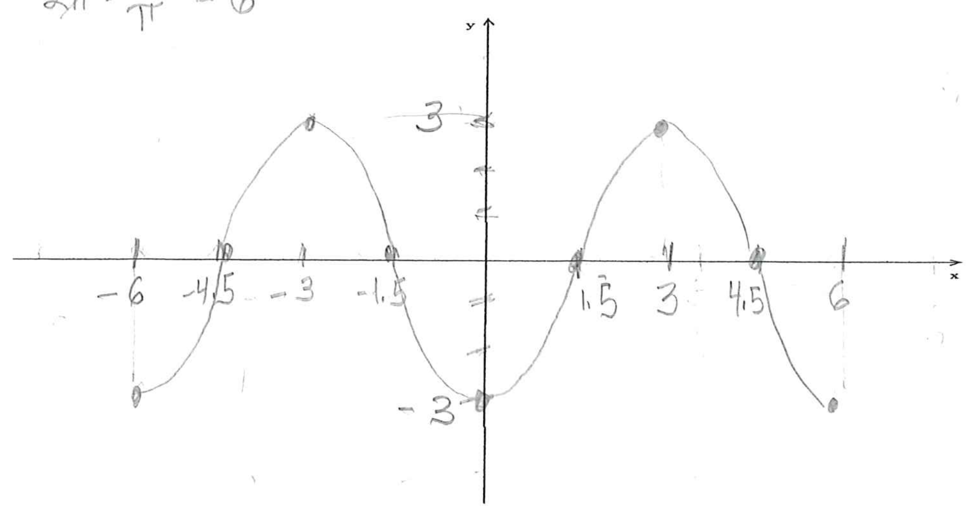
Amplitude: $|-3| = 3$

Period: $T = 6$

Midline: $y = 0$

Domain: $(-\infty, \infty)$

Range: $[-3, 3]$



2. (4 pts) In the functions $y = A\sin(B(x - x_0)) + D$ or $y = A\cos(B(x - x_0)) + D$
(circle the correct answer)

(a) The period is B | the period is A | $\boxed{\text{the period is } \frac{2\pi}{B}}$ | the period is D

(b) The amplitude is D | the amplitude is A | the amplitude is B | $\boxed{\text{the amplitude is } |A|}$

(c) $\boxed{\text{The midline is y = D}}$ | the midline is y = A | the midline is $y = x_0$ | the midline is y = B

(d) The phase shift is A | $\boxed{\text{the phase shift is } x_0}$ | the phase shift is D | the phase shift is B

3. (6 pts) (a) Find **all** x-values that make sine zero; i.e., solve sin(x) = 0.

List format: $x = 0, \pi, 2\pi, 3\pi, \ldots$
$-\pi, -2\pi, -3\pi, \ldots$

Formula (generative format): +1 $x = k\pi,\ k \in \mathbb{Z}$

(b) Find **all** x-values that make cosine zero; i.e., solve cos(x) = 0.

List format: $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$
$-\frac{\pi}{2}, -\frac{3\pi}{2}, -\frac{5\pi}{2}, \ldots$

Formula (generative format): +1 $x = \frac{(2k+1)\pi}{2},\ k \in \mathbb{Z}$

4. (3 pts) Vertical asymptotes are created by division by zero.

5. (12 pts) (a) Rewrite $y = \tan x$ using the Ratio Identity (i.e., in terms of sine and cosine).

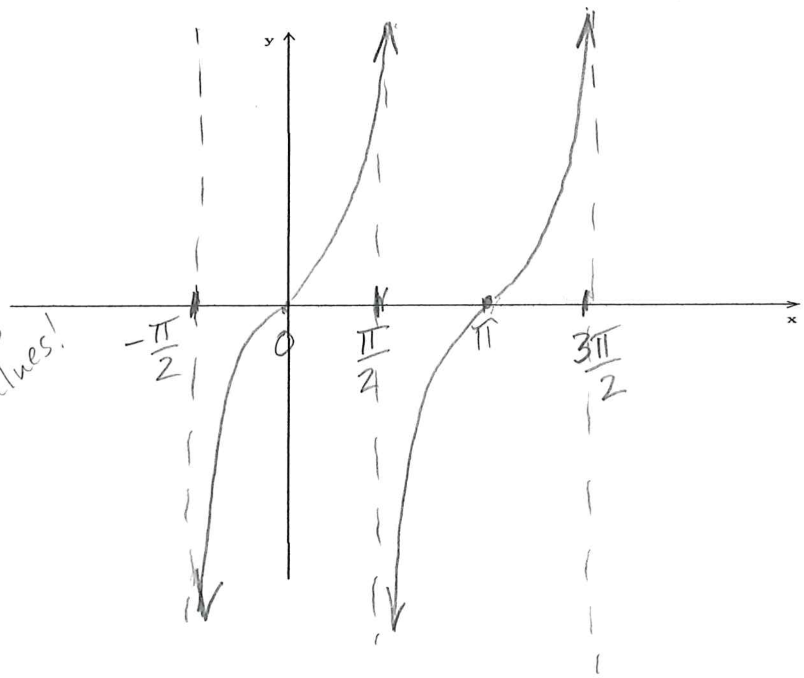
2 $y = \tan x = \frac{\sin(x)}{\cos(x)}$

2 (b) For what x-values will the graph of $y = \tan x$ have a Vertical Asymptote?
Give **all** possible values.

See #3b!
$x = \pi/2, 3\pi/2, \ldots$
$-\pi/2, -3\pi/2, \ldots$

6 (c) Graph 2 periods of $y = \tan x$. What is the period of this function? $\boxed{T = \pi}$

For full credit, the x-axis must be labeled with all relevant values (asymptotes).



omit

What is the domain of $y = \tan x$?

Domain: $\left\{ x \mid x \neq \frac{(2k+1)\pi}{2} \right\}$ ← ok to list values!

What is the range of $y = \tan x$?

2

Range: $(-\infty, \infty)$

6. (8 pts) Fill in the number of the graph that matches each equation:

A.) $y = 2\sin x$ ____(iii)____

amp

B.) $y = \sin(2x)$ ____(ii)____

$B=2$ so $T = \frac{2\pi}{2} = \pi$

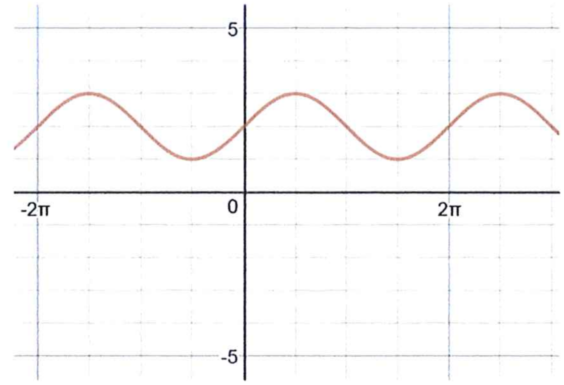
C.) $y = \sin(x) + 2$ ____(i)____

midline

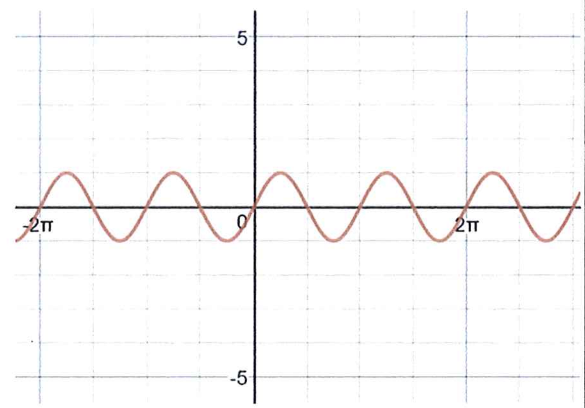
D.) $y = \sin(x+2)$ ____(iv)____

phase shift, left $\underline{\underline{2}}$

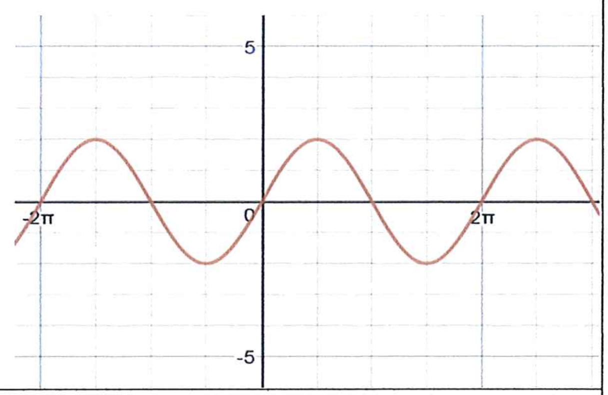
(i)



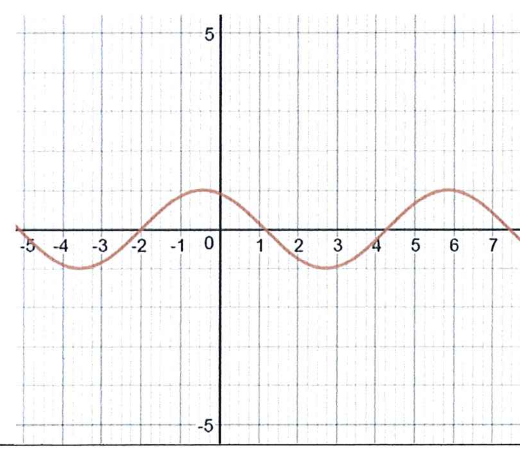
(ii)



(iii)



(iv)



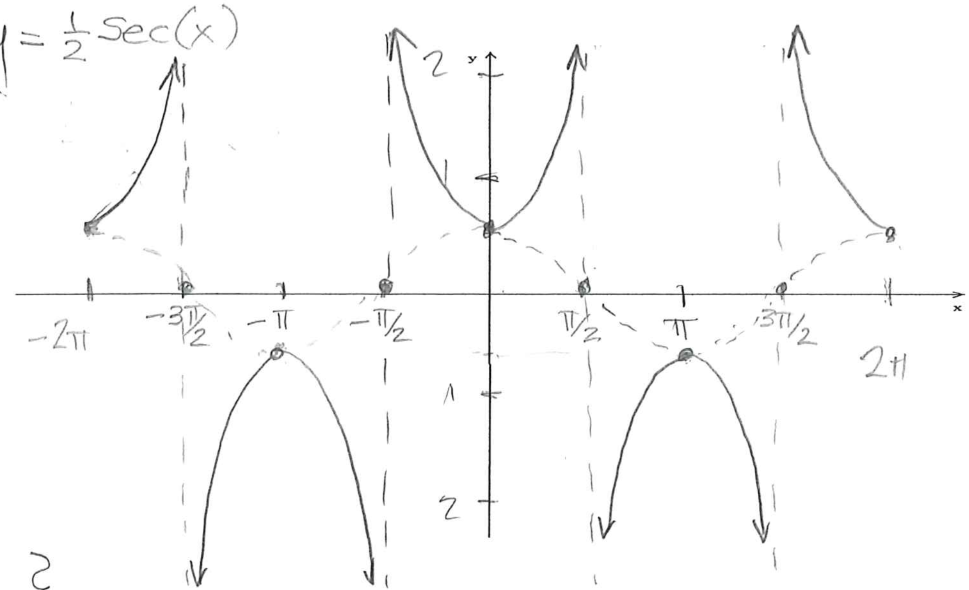
7. (8 points) Graph $y = \frac{1}{2}\sec(x)$ from $x = -2\pi$ to $x = 2\pi$. Clearly label the axes with key values.

Work off of $y = \frac{1}{2}\cos(x)$

Amp $= \frac{1}{2}$

2 $y = \frac{1}{2}\sec(x)$

4



2 Period: 2π

Domain: $\{x \mid x \neq \frac{(2k+1)\pi}{2}\}$

2 Range: $(-\infty, -\frac{1}{2}] \cup [\frac{1}{2}, \infty)$

8. (4 points) Determine the phase (horizontal) shift of the given function $y = \sin(\pi x - 3\pi)$. You do not have to graph!

$y = \sin(\pi(x-3))$

phase shift

Phase shift:

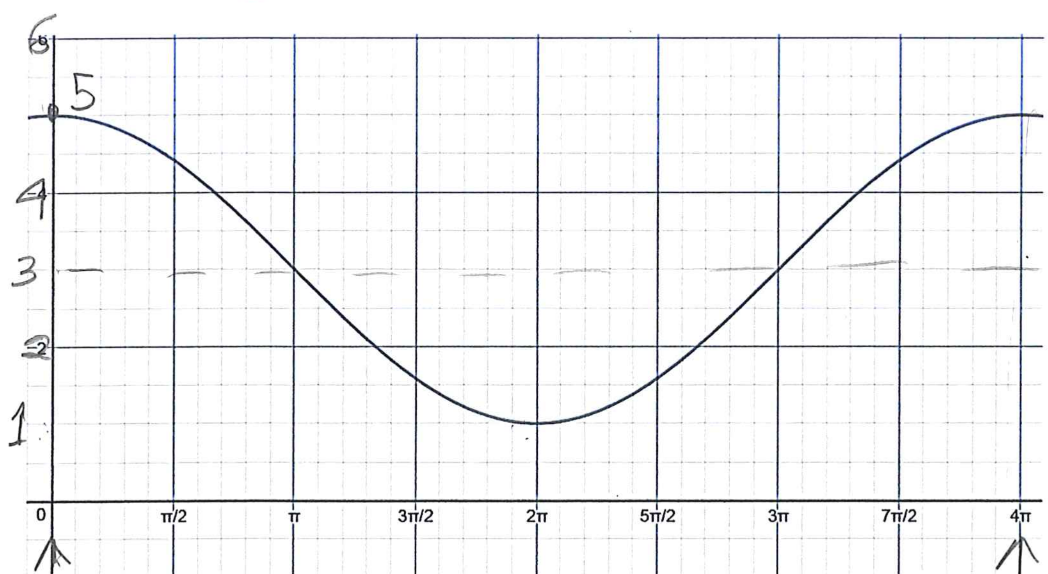
$x_0 = 3$

so Right 3 units

ok: $+3$

$\frac{1}{2}$ credit: 3π

9. (6 pts) Determine the equation of the function shown in the graph.



0 ← period → 4π

$T = 4\pi$

$B = \frac{2\pi}{4\pi} = \frac{1}{2}$

no phase shift if we use Cosine!

Midline: $y = 3$

Amp $= 2$

$\boxed{y = 2\cos\left(\frac{1}{2}x\right) + 3}$

10. (8 pts) A weight attached to the end of a long spring that is bouncing up and down next to a table. As it bounces, its distance above and below the table varies sinusoidally. Assume the table top is level with the midline of the bouncing weight.

The weight is 4 inches above the table top at its highest point and 4 inches below the table top at its lowest point. Assume the weight is at its highest point at t = 0 seconds and its lowest point at t = 1.5 seconds.

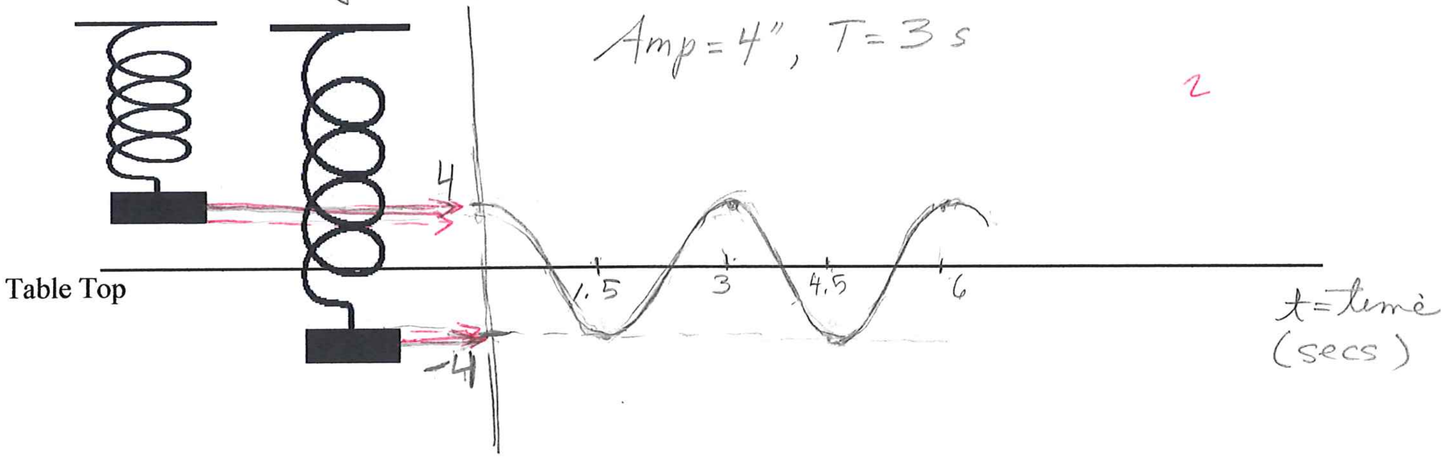
(a) Sketch a sinusoidal graph next to the picture of the weight, showing its position relative to the table top as a function of time.

Mention: LABEL axes!

y = displacement (inches)

Amp = 4", T = 3 s

2



6 (b) Write a cosine function to model the position of the weight relative to the table top.

$T = 3$ secs

$B = \frac{2\pi}{T} = \frac{2\pi}{3}$

$A = 4$ inches

$D = 0, t_0 = 0$

$$\boxed{y = 4\cos\left(\frac{2\pi}{3}t\right)}$$

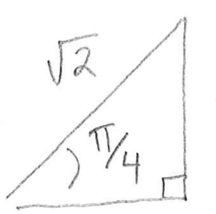
y = position (in)
t = time (s)

ok: $y = 4\left(\frac{2\pi}{3}x\right)$

11. (8 pts) Evaluate without using a calculator. For credit, you must sketch a right triangle. Write your answer in radians.

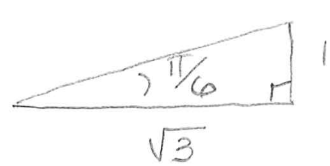
(a) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \underline{\frac{\pi}{4}}$

an angle whose sine is $\frac{1}{\sqrt{2}}$



(b) $\arctan\left(\frac{1}{\sqrt{3}}\right) = \underline{\frac{\pi}{6}}$

an angle whose tangent is $\frac{1}{\sqrt{3}}$



12. (4 pts) Explain why it isn't possible to find $\sin^{-1}\left(\frac{3}{2}\right)$. Include the sketch of a right triangle in your explanation

$\sin^{-1}\left(\frac{3}{2}\right)$ is asking for "an angle whose sine is $\frac{3}{2}$", so the hypotenuse is 2 and the opposite leg is 3. But the hypotenuse is always longer than the legs so this is impossible!

13. (6 pts) The grade of a road is 7%. This means for every horizontal distance of 100 feet on the road, the vertical rise is 7 feet. Find the angle the road makes with the horizontal in both radians and degrees.

Sketch:

$\tan\theta = \frac{7}{100} = .07$

$\theta = \tan^{-1}(.07)$

Angle (radians): .069886 or about .07! Angle (degrees): 4.004° or about 4°

14. (3 pts) Rewrite the following without -x in the argument:

$\sin(-x)\tan(-x) + \cos(-x)$

$-\sin(x) \cdot -\tan(x) + \cos(x)$

$= \boxed{\sin(x)\tan(x) + \cos(x)}$

15. (6 pts)

4 (a) Use the fact the cosine is an even function to prove secant is an even function; i.e., show that

$\sec(-x) = \sec(x)$

$\sec(-x) = \frac{1}{\cos(-x)}$) Because cosine is an even function

$= \frac{1}{\cos(x)}$

$= \sec(x)$

2 (b) Because $y = \sec(x)$ is an even function, we know its graph will have what type of symmetry?

Symmetry with respect to the y-axis.