Math 229: Test 2 (Wright, Spring 2019) **Name:** KEY
(100 points)

Please do your work in a well-organized manner. Credit is based on the amount of correct work shown, not just on the final answer. Give exact answers where asked for and use proper notation. Only scientific calculators are allowed on the exam.

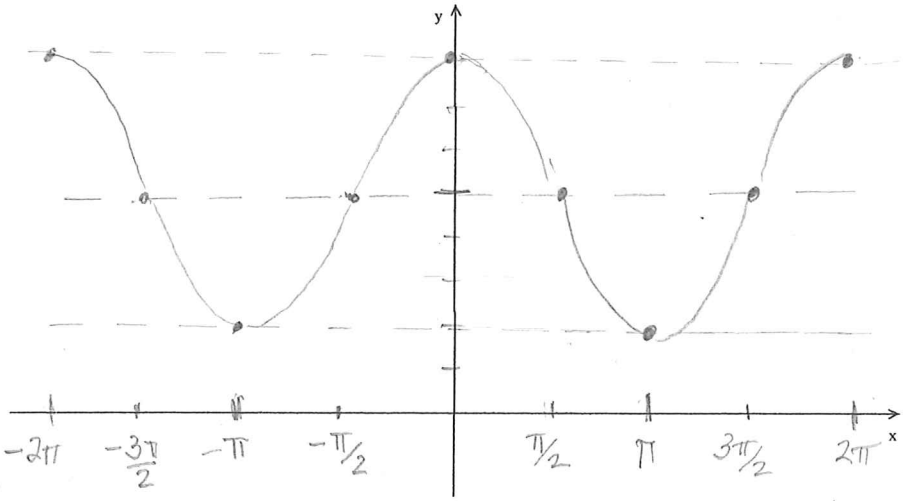
1. (10 pts) Determine the amplitude, period, and equation for the midline for the given function. Then sketch two full periods (one on each side of the y-axis) of the function. For full credit, the x-axis and y-axis must be well-labeled with all relevant values.

$y = 3\cos(x) + 5$

Midline equation: $y = 5$

Amplitude: 3

Period: 2π



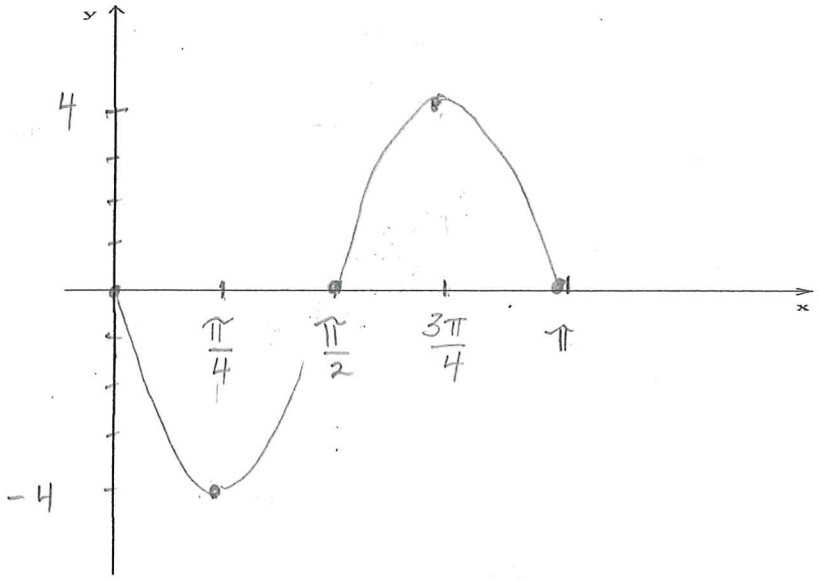
2. (10 pts) Sketch one full period of the function. For full credit, the x-axis and y-axis must be labeled with all relevant values.

$y = -4\sin(2x)$

Midline: $y = 0$

Amplitude: 4

Period: $T = \frac{2\pi}{2} = \pi$

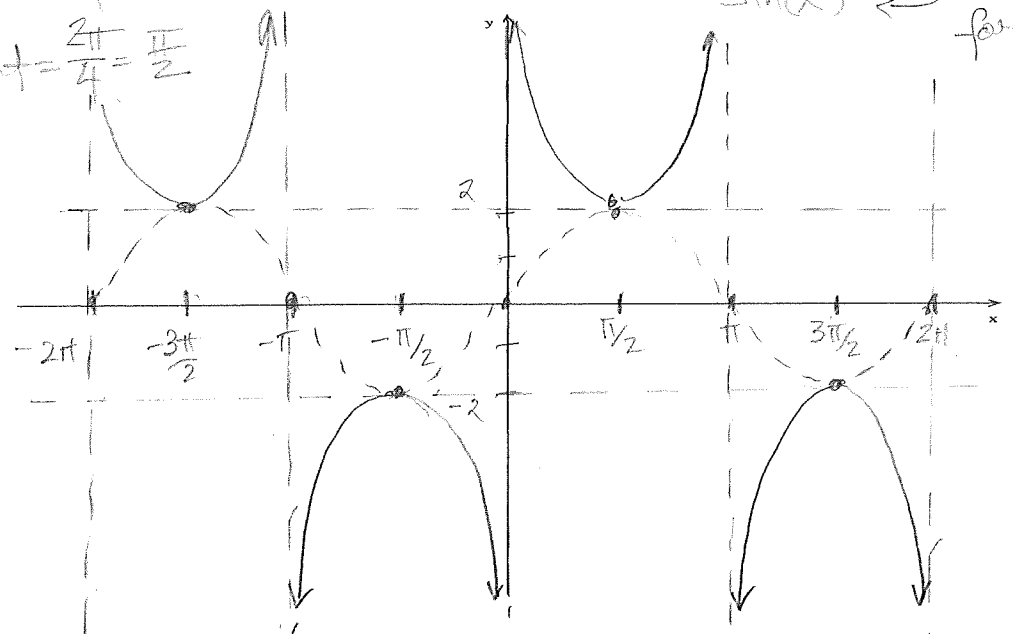


reflection: 3pts
amp positive 1pt
period/increased graph 3pt

3. (3 pts) Vertical Asymptotes are found where an x-value causes division by zero in a function.

4. (12 pts)
(a) Sketch two periods (one on each side of the y-axis) of $y = 2\csc(x)$. $= \frac{2}{\sin(x)}$ ← $\sin(x) = 0$ for $x = k\pi$

$T = 2\pi$
increment $= \frac{2\pi}{4} = \frac{\pi}{2}$



(b) List ALL x- values where $y = \csc(x)$ is undefined. (Hint: Reciprocal Identity)

$y = \csc(x) = \frac{1}{\sin(x)}$ so undefined when $\sin(x) = 0$

$x = 0, \pi, 2\pi, \ldots$
$-\pi, -2\pi, \ldots$ OR $x = k\pi, k \in \mathbb{Z}$

(c) Give the domain and range of the function (any format for domain and range is fine):

Domain: $\{x \mid x \neq k\pi, k \in \mathbb{Z}\}$ (or $x \neq 0, \pi, 2\pi, \ldots$ and $x \neq -\pi, -2\pi, \ldots$)

Range: $(-\infty, -2] \cup [2, \infty)$

ok
$y = \csc(x)$
or $y = 2\csc(x)$

5. (6 pts) (a) Rewrite $y = \tan x$ using the Ratio Identity (i.e., in terms of sine and cosine).

$y = \tan x = \frac{\sin(x)}{\cos(x)}$

(b) The graph of $y = \tan x$ will have a Vertical Asymptote when $\cos(x) = 0$

(c) List ALL the x-values where tan(x) has a Vertical Asymptote.

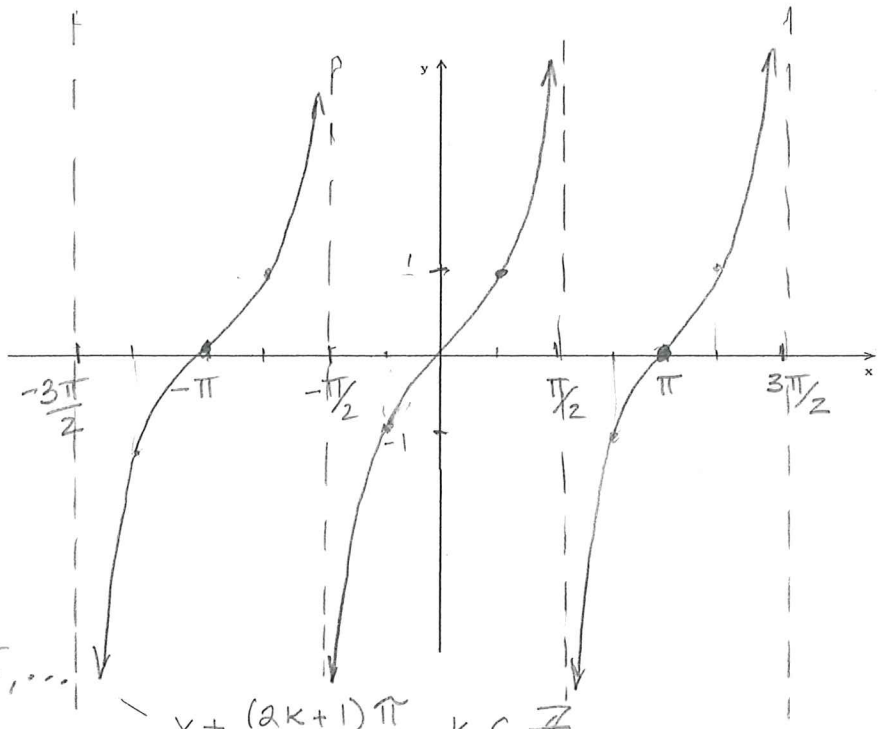
$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots$
and $x = -\frac{\pi}{2}, -\frac{3\pi}{2}, \ldots$

OR $x = \frac{(2k+1)\pi}{2}$ $k \in \mathbb{Z}$

OR $x = \frac{\pi}{2} + k\pi$ $k \in \mathbb{Z}$

6. (10 pts)

3 (a) Graph 3 periods of $y = \tan(x)$.
For full credit, the x-axis must be labeled with all relevant values.



3 (b) What is the period of $\tan(x)$?

$T = \pi$

2 + 2 (c) What is the domain and the range of $y = \tan x$?

Domain: $x \neq \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$ $x \neq -\pi/2, -3\pi/2, \ldots$ — $x \neq \frac{(2k+1)\pi}{2}$, $k \in \mathbb{Z}$

Range: $(-\infty, \infty)$

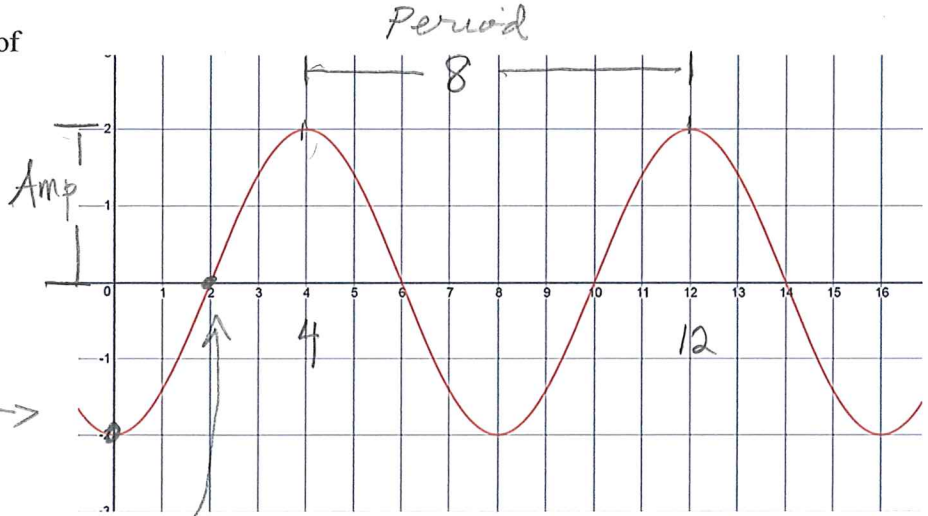
7. (4 points) Determine the phase (horizontal) shift of the function $y = \sin(\frac{\pi}{6}x + \pi)$.
You do not have to graph!

Factor $\frac{\pi}{6}$ out of $\frac{\pi}{6}x + \pi$: $\frac{\pi}{6}x + \pi = \frac{\pi}{6}(x + 6)$

Phase shift is -6 (6 units left)

Alternative method:
Solve $\frac{\pi}{6}x + \pi = 0$ to get $x = -6$.

8. (9 pts) Determine amplitude and period of the function shown in the graph:



1.5 Period: 8 units

1.5 Amplitude: 2 units

$T = 8 \Rightarrow B = \frac{2\pi}{8} = \frac{\pi}{4}$

3 (a) Write a formula for the graph that uses **cosine**.

flipped (reflected)

$y = -2\cos(\frac{\pi}{4}x)$

also $y = 2\cos(\pi/4(x-4))$

3 (b) Write a formula for the graph that uses **sine**.

phase shift of 2 for sine

$y = 2\sin(\frac{\pi}{4}(x-2))$

9. (6 pts) State whether each function is even, odd, or neither and what type of symmetry the graph has:

Function	Even or Odd	Type of Symmetry
$y = \sin(x)$	Odd	origin; 180° rotational symmetry
$y = \cos(x)$	Even	y-axis
$y = \tan(x)$	Odd	origin; 180° rotational symmetry

10. (2 pts) If $\cos(x) = 0.4$, then $\cos(-x) =$ 0.4 since $\cos(-x) = \cos(x) = .4$

11. (2 pts) If $\sin(x) = -0.2$, then $\sin(-x) =$ 0.2 since $\sin(-x) = -\sin(x)$
$= -(-.2)$
$= .2$

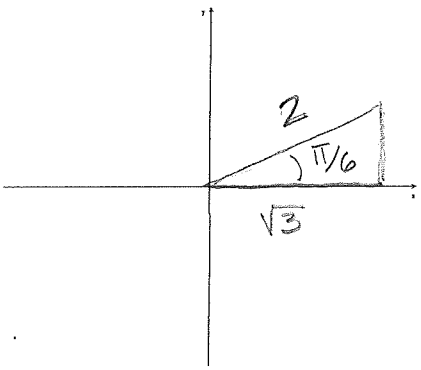
12. (2 pts) True or False: $\cos(-x)\sin(-x) = -\cos(x)\sin(x)$

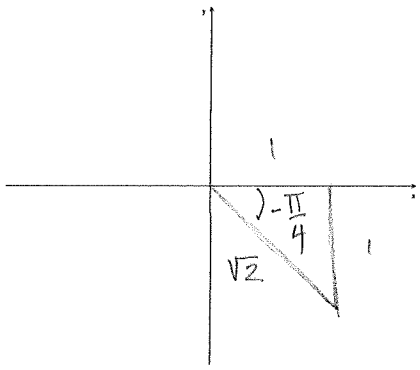
$\cos(-x) \cdot \sin(-x) = \cos(x) \cdot (-\sin(x)) = -\cos(x)\sin(x)$

13. (4 pts) Evaluate without using a calculator. For credit, sketch the angle and the reference triangle. Give the exact answer in RADIANS.

(a) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) =$ $\pi/6$

(b) $\arcsin\left(-\frac{1}{\sqrt{2}}\right) =$ $-\frac{\pi}{4}$





14. (4 pts) Use your calculator to evaluate each of the following.
Give your answer in DEGREES, to the nearest 10th of a degree. (one decimal place)

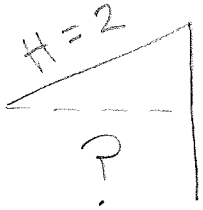
(a) $\arccos(-.9938) =$ 173.6°

(b) $\tan^{-1}(10) =$ 84.3°

15. (4 pts) Explain why $\sin^{-1}\left(\frac{3}{2}\right)$ is undefined. Include a sketch of a triangle in your answer

$\theta = \sin^{-1}\left(\frac{3}{2}\right)$

$\Rightarrow \sin\theta = \frac{3}{2}$



doesn't work!

The opposite side of a right triangle can't be greater than the hypotenuse.

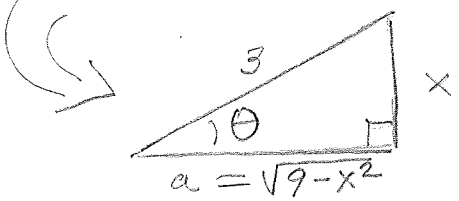
16. (6 pts) Find an algebraic expression in terms of x for $\cos\left(\sin^{-1}\left(\frac{x}{3}\right)\right)$. For full credit, include the sketch of a well-labeled triangle.

$\cos\left(\sin^{-1}\left(\frac{x}{3}\right)\right)$

$= \cos(\theta) =$ _____?

$\theta = \sin^{-1}\left(\frac{x}{3}\right)$

$\sin\theta = x/3$



$a^2 + x^2 = 3^2$

$a^2 = 9 - x^2$

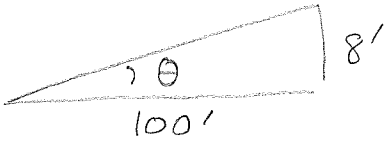
$a = \sqrt{9-x^2}$

$\Rightarrow \boxed{\cos\left(\sin^{-1}\left(\frac{x}{3}\right)\right) = \frac{\sqrt{9-x^2}}{3}}$

17. (6 pts) If the grade of a road is 8%, it means that for every horizontal distance of 100 feet, the vertical rise is 8 feet.

Find the angle (in degrees) that a road with 8% grade makes with the horizontal.

Sketch: (Not to scale! :)



$\tan\theta = \frac{8}{100} = .08$

$\theta = \tan^{-1}(.08) = 4.57^\circ$

number of decimal places will vary!