

**Math 229: Test 3** (Wright, Fall 2018)  
(100 points)

Name: \_\_\_\_\_

*Please do your work in a well-organized manner. Credit is based on the amount of correct work shown, not just on the final answer. Give exact answers where asked for and use proper notation. Only scientific calculators are allowed on the exam.*

1. (40 pts) Solve each of the following equations. Give the general solution in each case.

(a)  $\sin(x) = -\frac{1}{2}$

(b)  $\tan(x) = 1$

(Hint for (c) and (d): Use Double Angle Identities!)

(c)  $\cos^2(x) - \sin^2(x) = 1$

(d)  $\sin(2x) - \sqrt{3}\sin(x) = 0$

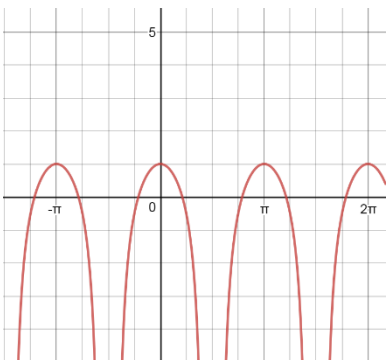
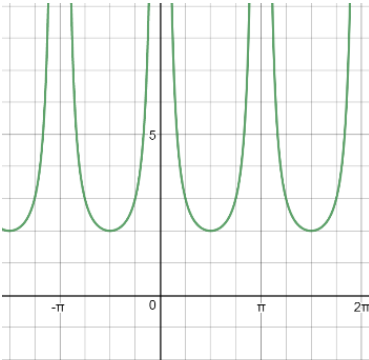
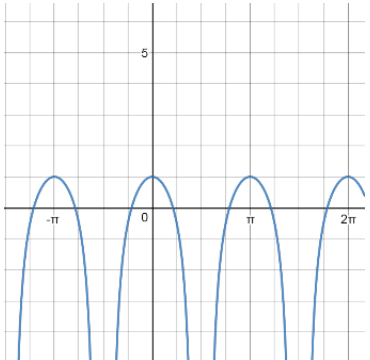
2. (10 pts) (a) Write the Sum of Angles Identity for cosine:

$$\cos(A + B) = \underline{\hspace{10em}}$$

(b) Show how the first Double Angle Identity,  $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$ , is derived from the Sum of Angles Identity.

(c) Show how  $\cos(2\theta) = 2\cos^2(\theta) - 1$  is derived from the identity above. (Hint: Pythagorean Identity.)

3. (10 pts) Consider each of the graphs shown. From the graphs determine which equations could be identities. You do NOT have to prove the identity!

<p>(a) <math>2 - \sin^2(x) - \sec^2(x) = \cos^2(x) - \tan^2(x)</math></p> <p>Possible identity? YES NO Can't tell</p>	<p>(b) <math>\csc^2(x)(1 + \sin^2(x)) = 2 - \sin^2(x) - \sec^2(x)</math></p> <p>Possible identity? YES NO Can't tell</p>	
<p><math>y = \cos^2(x) - \tan^2(x)</math></p> 	<p><math>y = \csc^2(x)(1 + \sin^2(x))</math></p> 	<p><math>y = 2 - \sin^2(x) - \sec^2(x)</math></p> 

4. (10 pts) Find the exact value of  $\sin\left(\frac{7\pi}{12}\right)$ . For credit, you must show work using the Sum of Angles Identity. Do not rationalize the denominator.

5. (10 pts) (a) Find the exact value of  $\tan\left(\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sin^{-1}\left(\frac{1}{2}\right)\right)$ ,  
using the identity  $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

Simplify your answer (you do not have to rationalize the denominator).

6. (10 pts) (a) If  $\theta$  is in Quadrant IV, fill in the numbers in degrees, choosing values from 0 to 360 degrees)

$$\underline{\hspace{2cm}} \leq \theta \leq \underline{\hspace{2cm}}$$

$$\underline{\hspace{2cm}} \leq \frac{\theta}{2} \leq \underline{\hspace{2cm}} \quad \text{Which quadrant is } \frac{\theta}{2} \text{ in? } \underline{\hspace{2cm}}$$

(b) Given  $\cos(\theta) = \frac{3}{5}$  and  $\theta$  is in Quadrant IV, find  $\sin\left(\frac{\theta}{2}\right)$ ,  $\cos\left(\frac{\theta}{2}\right)$ ,  $\tan\left(\frac{\theta}{2}\right)$ .

Be careful about the signs on your answers! Simplify all fractions.

For credit, show work using these Identities:  $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$        $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$

7. (10 pts) Prove/verify the identity. For full credit, make sure you show each step of the proof!

$$\cos(2x) = \frac{1 - \tan^2(x)}{1 + \tan^2(x)}$$