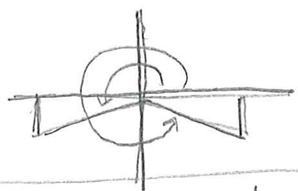
Math 229: Test 3 (Wright, Fall 2018) **Name:** Key
(100 points)

Please do your work in a well-organized manner. Credit is based on the amount of <u>*correct*</u> *work shown, not just on the final answer. Give exact answers where asked for and use proper notation. Only* <u>*scientific calculators*</u> *are allowed on the exam.*

1. (40 pts) Solve each of the following equations. Give the <u>general solution</u> in each case.

(a) $\sin(x) = -\frac{1}{2}$ QIII QIV

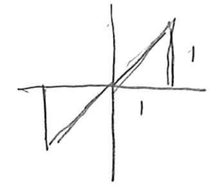
$\theta_{Ref} = \frac{\pi}{6}$



$$\boxed{\begin{aligned} x &= \frac{7\pi}{6} + 2\pi k \\ x &= \frac{11\pi}{6} + 2\pi k \end{aligned}}$$

$x = -\frac{\pi}{6} + 2\pi k$ also ok

(b) $\tan(x) = \frac{1}{1}$ QI QIII



$\theta_{Ref} = \pi/4$

best ans:

$$\boxed{x = \frac{\pi}{4} + \pi k}$$

OR

$x = \frac{\pi}{4} + 2\pi k$

$x = \frac{3\pi}{4} + 2\pi k$

ok

note: this isnt necessary to include since the period of tan(x) is π, but ok if you did!

(Hint for (c) and (d): Use Double Angle Identities!)

(c) $\cos^2(x) - \sin^2(x) = 1$

$\cos(2x) = 1$ ← This is a value for an axis angle

$\theta_{Ref} = 0$

$$\frac{2x}{2} = \frac{0 + 2\pi k}{2}$$

$\boxed{x = \pi k}$ best answer

Alternate solution method

$\cos^2(x) - \sin^2(x) = 1$

$2\cos^2(x) - 1 = 1$

$2\cos^2(x) = 2$

$\cos^2(x) = 1$

$\cos(x) = \pm\sqrt{1} = \pm 1$

$\cos(x) = 1$ $\quad$ $\cos(x) = -1$

$x = 0 + 2\pi k$ $\quad$ $x = \pi + 2\pi k$

$x = 2\pi k$

ok

(d) $\sin(2x) - \sqrt{3}\sin(x) = 0$

$2\sin(x)\cos(x) - \sqrt{3}\sin(x) = 0$

factor!

$\sin(x)\left[2\cos(x) - \sqrt{3}\right] = 0$

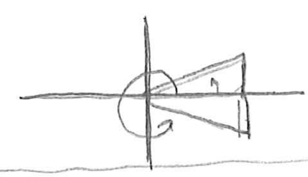
$\sin(x) = 0$ OR $2\cos(x) - \sqrt{3} = 0$

$\boxed{x = k\pi}$

$\cos(x) = \frac{\sqrt{3}}{2}$

QI, QIV

$\theta_{Ref} = \frac{\pi}{6}$ (see (a))



$$\boxed{\begin{aligned} x &= \frac{\pi}{6} + 2\pi k \\ x &= \frac{11\pi}{6} + 2\pi k \end{aligned}}$$

2. (10 pts) (a) Write the Sum of Angles Identity for cosine:

$\cos(A+B) =$ $\cos A \cos B - \sin A \sin B$

(b) Show how the first Double Angle Identity, $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$, is derived from the Sum of Angles Identity.

$$\cos(2\theta) = \cos(\theta + \theta)$$
$$= \cos(\theta)\cos(\theta) - \sin(\theta)\sin(\theta)$$
$$= \cos^2(\theta) - \sin^2(\theta) \quad \blacksquare$$

(c) Show how $\cos(2\theta) = 2\cos^2(\theta) - 1$ is derived from the identity above. (Hint: Pythagorean Identity.)

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$
$$= \cos^2(\theta) - (1 - \cos^2(\theta))$$
$$= \cos^2(\theta) - 1 + \cos^2(\theta)$$
$$= 2\cos^2(\theta) - 1 \quad \blacksquare$$

$\sin^2(\theta) = 1 - \cos^2(\theta)$

3. (10 pts) Consider each of the graphs shown. From the graphs determine which equations could be identities. You do NOT have to prove the identity!

(a) $2 - \sin^2(x) - \sec^2(x) = \cos^2(x) - \tan^2(x)$

Possible identity? YES NO Can't tell

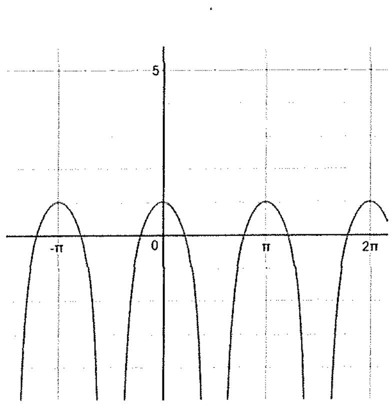
Graphs match!

(b) $\csc^2(x)\left(1 + \sin^2(x)\right) = 2 - \sin^2(x) - \sec^2(x)$

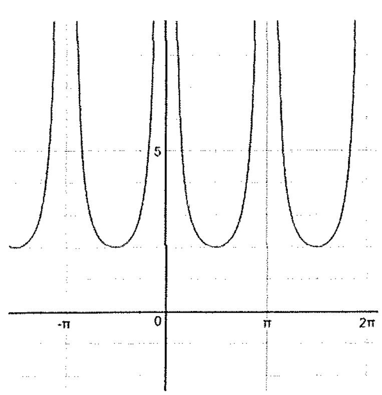
Possible identity? YES NO Can't tell

Graphs don't match!

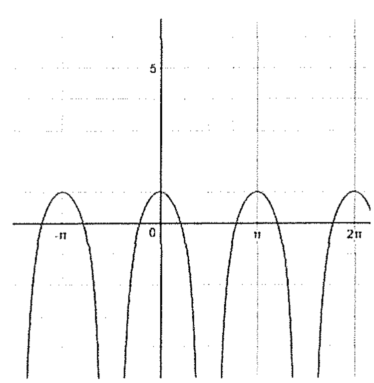
$y = \cos^2(x) - \tan^2(x)$



$y = \csc^2(x)\left(1 + \sin^2(x)\right)$

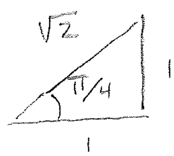


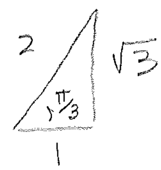
$y = 2 - \sin^2(x) - \sec^2(x)$



4. (10 pts) Find the exact value of $\sin\left(\frac{7\pi}{12}\right)$. For credit, you must show work using the Sum of Angles Identity. Do not rationalize the denominator.

$$\frac{7\pi}{12} = \frac{3\pi}{12} + \frac{4\pi}{12} = \frac{\pi}{4} + \frac{\pi}{3}$$





$$\sin\left(\frac{7\pi}{12}\right)$$

$$= \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$$

$$= \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) + \sin\left(\frac{\pi}{3}\right)\cdot\cos\left(\frac{\pi}{4}\right)$$

$$= \frac{1}{\sqrt{2}}\left(\frac{1}{2}\right) + \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2}\right)$$

$$\boxed{= \frac{1+\sqrt{3}}{2\sqrt{2}}}$$

5. (10 pts) (a) Find the exact value of $\tan\left(\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sin^{-1}\left(\frac{1}{2}\right)\right)$,

using the identity $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

Simplify your answer (you do not have to rationalize the denominator).

$$\tan\left(\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sin^{-1}\left(\frac{1}{2}\right)\right)$$

$$= \tan(A + B)$$

$$= \tan\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$$

$$= \frac{\tan\left(\frac{\pi}{4}\right) + \tan\frac{\pi}{6}}{1 - \tan\left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{6}\right)}$$

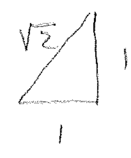
$$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1\cdot\frac{1}{\sqrt{3}}} = \frac{\left(1 + \frac{1}{\sqrt{3}}\right)\sqrt{3}}{\left(1 - \frac{1}{\sqrt{3}}\right)\sqrt{3}}$$

$$\boxed{= \frac{\sqrt{3}+1}{\sqrt{3}-1}}$$

$A = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\cos(A) = \frac{1}{\sqrt{2}}$

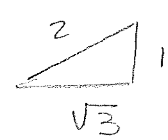
$A = \frac{\pi}{4}$



$B = \sin^{-1}\left(\frac{1}{2}\right)$

$\sin B = \frac{1}{2}$

$B = \frac{\pi}{6}$



6. (10 pts) (a) If θ is in Quadrant IV, fill in the numbers in degrees, choosing valuesfrom 0 to 360 degrees)

$270° \le \theta \le 360°$

$135° \le \frac{\theta}{2} \le 180°$ Which quadrant is $\frac{\theta}{2}$ in? QII

(b) Given $\cos(\theta) = \frac{3}{5}$ and θ is in Quadrant IV, find $\sin\left(\frac{\theta}{2}\right)$, $\cos\left(\frac{\theta}{2}\right)$, $\tan\left(\frac{\theta}{2}\right)$.

Be careful about the signs on your answers! Simplify all fractions.

For credit, show work using these Identities: $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$ $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$

QII

$$\sin\left(\frac{\theta}{2}\right) = +\sqrt{\frac{1-\frac{3}{5}}{2}} = \sqrt{\frac{\frac{2}{5}}{2}} = \sqrt{\frac{1}{5}} = \frac{1}{\sqrt{5}}$$

$$\boxed{\sin\left(\frac{\theta}{2}\right) = \frac{1}{\sqrt{5}}}$$

QII

$$\cos\left(\frac{\theta}{2}\right) = -\sqrt{\frac{1+\frac{3}{5}}{2}} = -\sqrt{\frac{\frac{8}{5}}{2}} = -\sqrt{\frac{4}{5}} = -\frac{2}{\sqrt{5}}$$

$$\boxed{\cos\left(\frac{\theta}{2}\right) = -\frac{2}{\sqrt{5}}}$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)} = \frac{\frac{1}{\sqrt{5}}}{\frac{-2}{\sqrt{5}}} = -\frac{1}{2}$$

$$\boxed{\tan\left(\frac{\theta}{2}\right) = -\frac{1}{2}}$$

7. (10 pts) Prove/verify the identity. For full credit, make sure you show each step of the proof!

$$\cos(2x) = \frac{1-\tan^2(x)}{1+\tan^2(x)}$$

$$\text{RHS}: \frac{1-\tan^2(x)}{1+\tan^2(x)} = \frac{\left[1-\frac{\sin^2(x)}{\cos^2(x)}\right]\cos^2(x)}{\left[1+\frac{\sin^2(x)}{\cos^2(x)}\right]\cos^2(x)}$$

$$= \frac{\cos^2(x)-\sin^2(x)}{\cos^2(x)+\sin^2(x)}$$

$$= \frac{\cos(2x)}{1} = \cos(2x) \blacksquare$$

$$= \text{LHS}$$

Note: Per Paris' advice, we could have swapped out $1+\tan^2(x)$ for $\sec^2(x)$ which would have saved us a step! :)