Math 229: Test 3 (Wright, Spring 2019) **Name:** KEY
(100 points)

Please do your work in a well-organized manner. Credit is based on the amount of correct work shown, not just on the final answer. Give exact answers where asked for and use proper notation. Only scientific calculators are allowed on the exam.

For each, $k \in \mathbb{Z}$

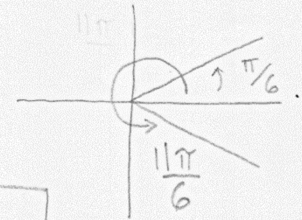
1. (40 pts) Solve each of the following equations. Give the general solution in each case.

10 (a) $2\cos(x) = \sqrt{3}$

Positive: QI QIV

$\cos(x) = \frac{\sqrt{3}}{2}$

$\theta_{Ref} = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$



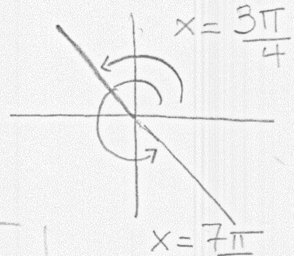
$$\boxed{\begin{aligned} x &= \frac{\pi}{6} + 2\pi k \\ x &= \frac{11\pi}{6} + 2\pi k \end{aligned}}$$

10 (b) $\tan(x) = -1$

Negative: QII QIV

$\theta_{Ref} = \tan^{-1}|-1| = \tan^{-1}(1) = \frac{\pi}{4}$

$x = \frac{3\pi}{4}$



$x = \frac{7\pi}{4}$

ok
$$\begin{aligned} x &= \frac{3\pi}{4} + 2\pi k \\ x &= \frac{7\pi}{4} + 2\pi k \end{aligned}$$

Best answer: $\boxed{x = \frac{3\pi}{4} + \pi k}$

(Hint for (c) and (d): Use Identities!)

8 (c) $\cos^2(x) + \sin^2(x) = \sin(x)$

$1 = \sin(x)$

By inspection: $\boxed{x = \frac{\pi}{2} + 2\pi k}$

12 (d) $\sin(2x) = \cos(x)$

$2\sin(x)\cos(x) - \cos(x) = 0$

$\cos(x)\left[2\sin(x) - 1\right] = 0$

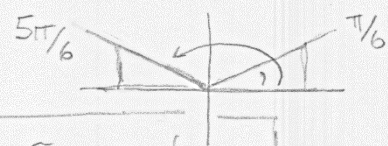
$\cos(x) = 0$

By inspection: $\boxed{x = \frac{\pi}{2} + \pi k}$

$2\sin(x) - 1 = 0$

$\sin(x) = \frac{1}{2}$

QI QII



$$\boxed{\begin{aligned} x &= \frac{\pi}{6} + 2\pi k \\ x &= \frac{5\pi}{6} + 2\pi k \end{aligned}}$$

4. (10 pts) Find the exact value of $\cos\left(\frac{5\pi}{12}\right)$. For credit, you must show work using the Sum of Angles Identity. You do not have to rationalize the denominator.

$$\frac{\pi}{6} = \frac{2\pi}{12} \qquad \frac{\pi}{4} = \frac{3\pi}{12} \qquad \frac{\pi}{3} = \frac{4\pi}{12}$$

$$\cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$$

$$= \cos\left(\frac{\pi}{6}\right) \cdot \cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}\right)$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} - \frac{1}{2} \cdot \frac{1}{\sqrt{2}}$$

$$= \boxed{\frac{\sqrt{3} - 1}{2\sqrt{2}}}$$

5. (10 pts) (a) Find the exact value of $\tan\left(\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right)$,

using the identity $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

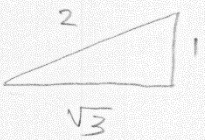
Simplify your answer (you do not have to rationalize the denominator).

$$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6} \qquad \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

$$\tan\left[\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right]$$

$$= \tan\left(\frac{\pi}{6} - \frac{\pi}{4}\right)$$

$$= \frac{\tan\left(\frac{\pi}{6}\right) - \tan\left(\frac{\pi}{4}\right)}{1 + \tan\left(\frac{\pi}{6}\right)\tan\left(\frac{\pi}{4}\right)}$$



$$= \frac{\frac{1}{\sqrt{3}} - 1}{1 + \frac{1}{\sqrt{3}}(1)}$$

2 points

$$= \boxed{\frac{1 - \sqrt{3}}{\sqrt{3} + 1}}$$

other form: $\dfrac{\sqrt{3} - 3}{3 + \sqrt{3}}$

2. (10 pts) (a) Write the Sum of Angles Identity for sine:

$\sin(A+B) =$ Sin(A)cos(B) + sin(B)cos(A)

(b) Show how the first Double Angle Identity, $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$, is derived from the Sum of Angles Identity.

$$\sin(2\theta) = \sin(\theta+\theta)$$
$$= \sin(\theta)\cos(\theta) + \sin(\theta)\cos(\theta)$$
$$= 2\sin(\theta)\cos(\theta) \quad \blacksquare$$

3. (10 pts) Consider the graphs shown. From the graphs determine whether the equation could be an identity.

(a)
$\csc^2(x) + \sec^2(x) - 2 = \sec(2x) - \sin^2(x)\cot^2(x)$

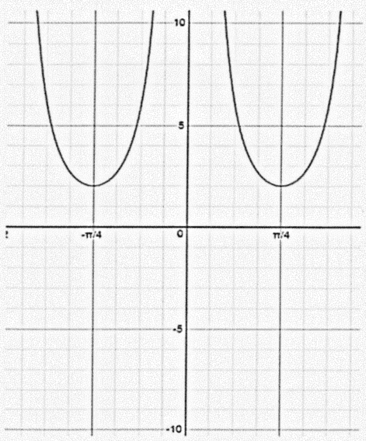
Possible identity? YES NO

(b)
$\csc^2(x) + \sec^2(x) - 2 = \cot^2(x) + \tan^2(x)$

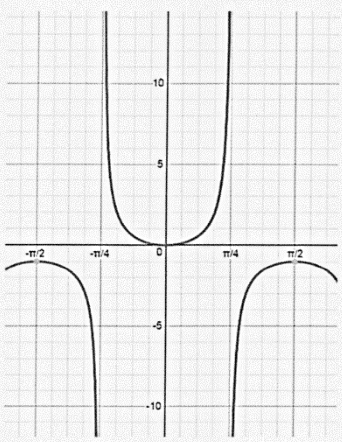
Possible identity? YES NO

Graphs for reference

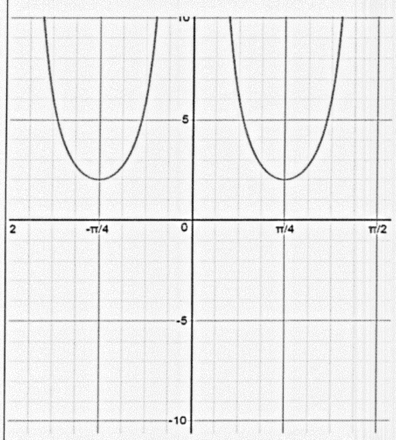
$y = \csc^2(x) + \sec^2(x) - 2$



$y = \sec(2x) - \sin^2(x)\cot^2(x)$



$y = \cot^2(x) + \tan^2(x)$



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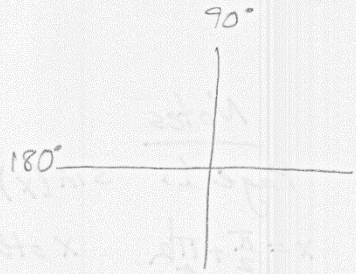
6. (10 pts) (a) If x is in Quadrant II, then $\underline{90^\circ \text{ or } \pi/2} \le x \le \underline{180^\circ \quad \pi}$



So, $\underline{45^\circ \text{ or } \pi/4} \le \dfrac{x}{2} \le \underline{90^\circ \quad \pi/2}$

Based on this, which quadrant is $\dfrac{x}{2}$ in? $\underline{QI}$

7 (b) Given $\boxed{\sec x = -4}$ and θ is in Quadrant II, find $\sin\left(\dfrac{x}{2}\right)$, $\cos\left(\dfrac{x}{2}\right)$, $\tan\left(\dfrac{x}{2}\right)$.

Be careful about the signs on your answers! Simplify all fractions.

For credit, show work using these Identities: $\sin\left(\dfrac{x}{2}\right) = \pm\sqrt{\dfrac{1-\cos x}{2}}$ $\quad\cos\left(\dfrac{x}{2}\right) = \pm\sqrt{\dfrac{1+\cos x}{2}}$

$\sec(x) = -4 \Rightarrow \cos(x) = -\dfrac{1}{4}$

$$\sin\left(\frac{x}{2}\right) = \overset{QI}{+}\sqrt{\frac{1-\left(-\frac{1}{4}\right)}{2}} = \sqrt{\frac{\frac{5}{4}}{2}} = \sqrt{\frac{5}{8}} \text{ OR } \frac{\sqrt{5}}{\sqrt{8}} \text{ OR } \frac{\sqrt{10}}{4}$$

any form is okay

$$\cos\left(\frac{x}{2}\right) = \overset{QI}{+}\sqrt{\frac{1+\left(-\frac{1}{4}\right)}{2}} = \sqrt{\frac{3/4}{2}} = \sqrt{\frac{3}{8}} = \frac{\sqrt{3}}{\sqrt{8}} = \frac{\sqrt{6}}{4}$$

$$\tan\left(\frac{x}{2}\right) = \frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)} = \frac{\frac{\sqrt{5}}{\sqrt{8}}}{\frac{\sqrt{3}}{\sqrt{8}}} = \boxed{\frac{\sqrt{5}}{\sqrt{3}}}$$

7. (10 pts) Prove/verify the identity. For full credit, make sure you show each step of the proof!

$$\frac{\cos(a+b)}{\cos(a)\cos(b)} = 1 - \tan(a)\tan(b)$$

$$\text{L.H.S.} = \frac{\cos(a+b)}{\cos(a)\cos(b)} = \frac{\cos(a)\cos(b) - \sin(a)\sin(b)}{\cos(a)\cos(b)}$$

$$= \frac{\cos(a)\cos(b)}{\cos(a)\cos(b)} - \frac{\sin(a)\sin(b)}{\cos(a)\cos(b)}$$

$$= 1 - \tan(a)\tan(b) \quad \blacksquare$$

$$= \text{R.H.S}$$