Math 229: Test 4 (In Class) **Name:** KEY

In-Class Test: ______________/70 Take-Home Test: ______________/30

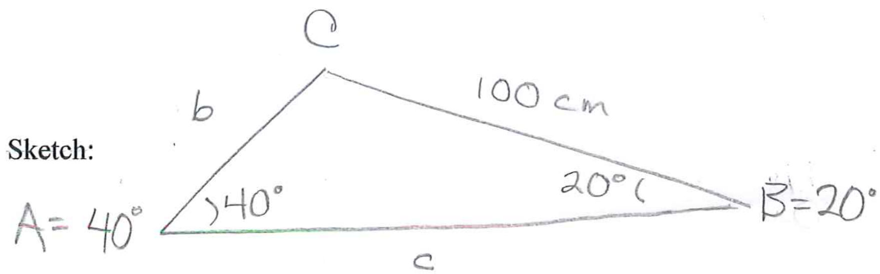
Please do your work in a well-organized manner. Credit is based on the amount of correct work shown, not just on the final answer. Use proper notation. Only *scientific calculators* are allowed on the exam.
Express answers to 3 significant figures (suggestion only, not required).

The following problems refer to a triangle ABC which has angles and/or sides as given. Solve for the indicated side or angle.

1. $A = 40^{\circ}$, a = 100 cm, and $B = 20^{\circ}$

1 (a) Sketch the triangle. (Angles A and B should be reasonable approximations.)

Sketch:



1 (b) Angle C is which? (circle the answer) ACUTE OBTUSE

2 (c) Use the Law of Sines to find b.

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin 20^{\circ}}{b} = \frac{\sin 40^{\circ}}{100}$$

$$\Rightarrow b = \frac{100 \sin 20^{\circ}}{\sin 40^{\circ}}$$

$$\boxed{b = 53.2\,\text{cm}}$$

3 (d) Find the other missing angles and sides of the triangle.

Note: You could also use the Law of Cosines to find c.

$$C = 180^{\circ} - 20^{\circ} - 40^{\circ}$$

$$\boxed{C = 120^{\circ}}$$

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin 120^{\circ}}{c} = \frac{\sin 40^{\circ}}{100}$$

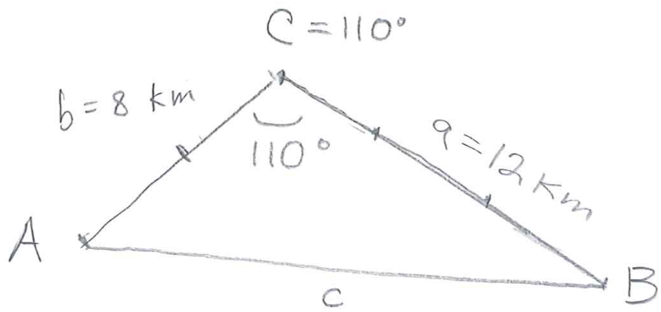
$$\Rightarrow c = \frac{100 \sin 120^{\circ}}{\sin 40^{\circ}}$$

$$\boxed{c = 135\,\text{cm}}$$

Right triangle -2

2. If a = 12 km, b = 8 km, and $C = 110^{\circ}$,

1 (a) Make a reasonable sketch of Triangle ABC:



3 (b) Use the Law of Cosines to find c.

$$c^2 = 12^2 + 8^2 - 2(12)(8)\cos 110^{\circ}$$

$$c^2 = 273.667$$

$$\boxed{c = 16.5 \text{ km}}$$

3 (c) Find the other missing angles (A and B) ← you can also use Law of Cosines

$$\boxed{\begin{array}{l} A = 43.0^{\circ} \\ B = 27.0^{\circ} \end{array}}$$

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin 110^{\circ}}{16.5} = \frac{\sin A}{12} \Rightarrow \sin A = \frac{12 \sin 110^{\circ}}{16.5}$$

$$\sin A = .6816$$

$$A = \sin^{-1}(.6816) = 43.0^{\circ}$$

↑ This will introduce rounding error! I used the entire value in calc.

3. (a) Use the Law of Sines to show that no triangle exists for which $A = 60^{\circ}$, a = 1 inch, and b = 3 inches.

2

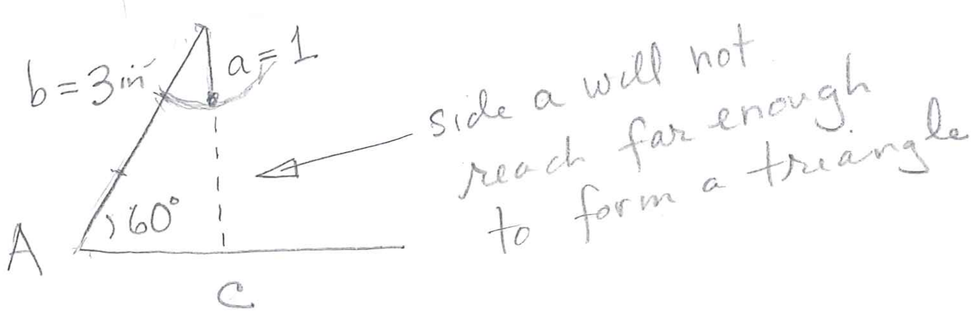
$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 60^{\circ}}{1} = \frac{\sin B}{3} \Rightarrow \sin B = 3 \sin 60^{\circ}$$

$$\sin B = 2.598 > 1$$

no such angle B exists!

2 (b) Make a sketch of the given sides and angle to illustrate why Triangle ABC doesn't exist.

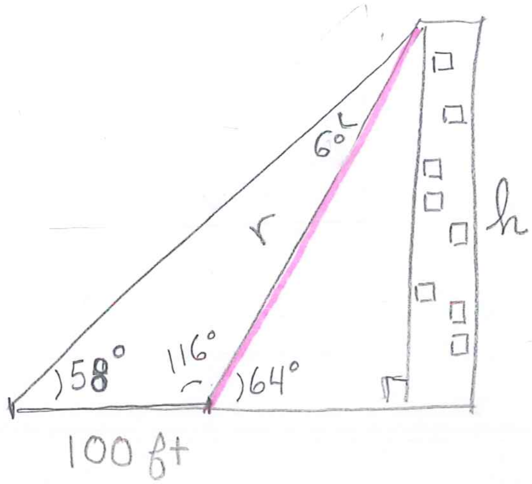


4. Faith, while practicing surveying techniques, finds the angle of elevation to the top of a building is 64°. She then moves 100 feet away from the building and notes that the angle of elevation is now 58°. Find the height of the building.

7

Include a detailed sketch with the solution.

Find pink side, r, then find h



$$\frac{\sin 6^{\circ}}{100} = \frac{\sin 58^{\circ}}{r}$$

$$r = \frac{100 \sin 58^{\circ}}{\sin 6^{\circ}} = \boxed{811 \text{ feet}}$$

$h = r \sin 64^{\circ}$

$h = 811 \sin 64^{\circ}$

$h = 729$ feet

(Same comment as before - use the unrounded version of r to get the most accurate answer.)

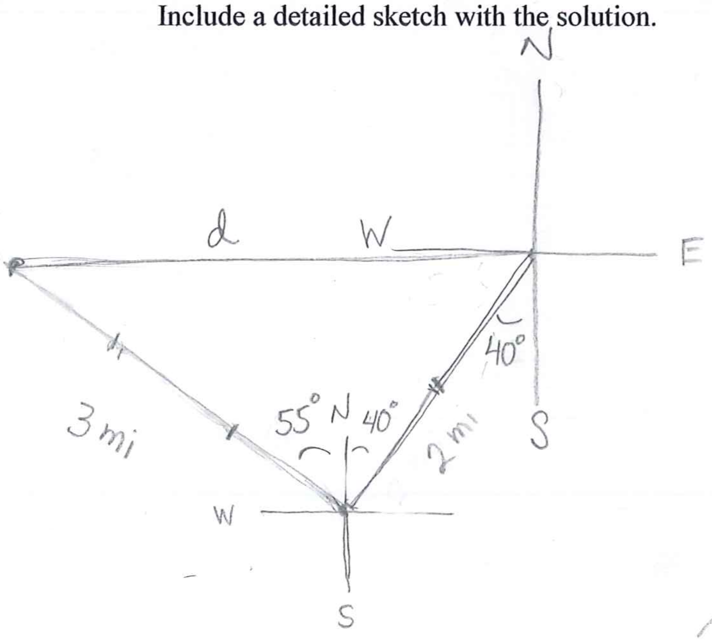
The building is 729 feet tall

5. Andrew is wandering in the desert. He walks 2 miles in the direction of $S40^{\circ}W$. He then turns and walks 3 miles in the direction of $N55^{\circ}W$. If he wants to walk directly back to his starting point, how far will he have to go?

7

Include a detailed sketch with the solution.

You could use vector addition here but it would be a looong and arduous approach!



Andrew will have to walk about 3.7 miles

Law of Cosines:

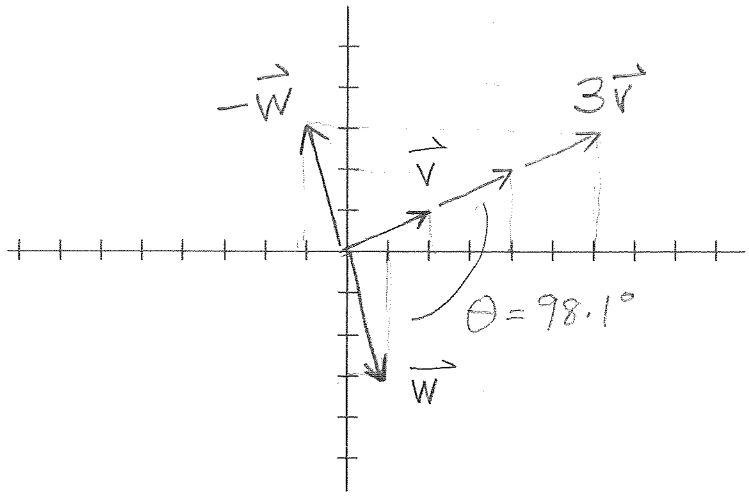
$d^2 = 3^2 + 2^2 - 2(3)(2)\cos 95^{\circ}$

$d^2 = 14.046$

$d = 3.7$ mi

6. Use the given vectors to do the following: $\vec{v} = 2\hat{i} + \hat{j}$ $\vec{w} = \hat{i} - 3\hat{j}$

2 (a) Graph both of these vectors.



1 (b) Graph $3\vec{v}$

$$3\vec{V} = 3\langle 2,1\rangle$$
$$= \langle 6,3\rangle$$

1 (c) Graph $-\vec{w}$

$$-\vec{W} = \langle -1, 3\rangle$$

2 (d) Find $4\vec{v} - 5\vec{w}$

$$4\langle 2,1\rangle - 5\langle 1,-3\rangle$$
$$= \langle 8,4\rangle + \langle -5, 15\rangle$$
$$= \boxed{\langle 3, 19\rangle}$$

2 (e) Find $\vec{v} \bullet \vec{w}$

$$\vec{V} \cdot \vec{W} = 2\cdot 1 + 1(-3)$$
$$= \boxed{-1}$$

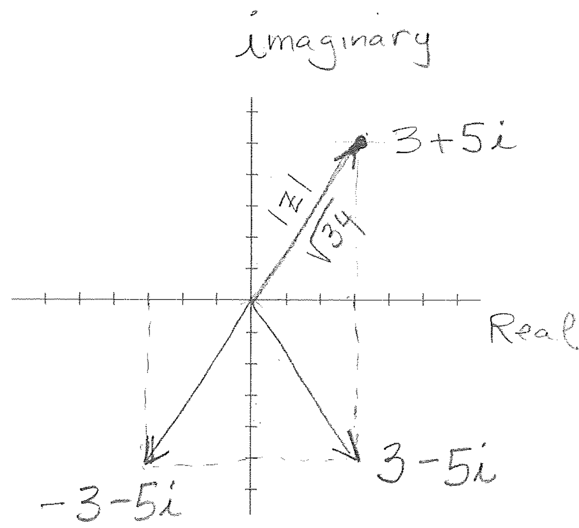
2 (f) Find $|\vec{v}|$ and $|\vec{w}|$

$$|\vec{V}| = \sqrt{2^2+1^2} = \sqrt{5}$$
$$|\vec{W}| = \sqrt{1^2+(-3)^2} = \sqrt{10}$$

3 (g) Find the angle between $\vec{v}$ and $\vec{w}$

$$\vec{V} \cdot \vec{W} = |\vec{V}|\,|\vec{W}| \cos\theta$$
$$-1 = \sqrt{5}\sqrt{10} \cos\theta$$
$$\frac{-1}{\sqrt{50}} = \cos\theta \rightarrow \theta = \cos^{-1}\left(\frac{-1}{\sqrt{50}}\right)$$
$$\boxed{\theta = 98.1°}$$

7. Given the complex number $z = 3 + 5i$



1 (a) Graph the complex number on the Complex Plane.

2 (b) Find the absolute value of z and indicate what part of the graph represents this.

$$|3+5i| = \sqrt{3^2+5^2} = \sqrt{34}$$

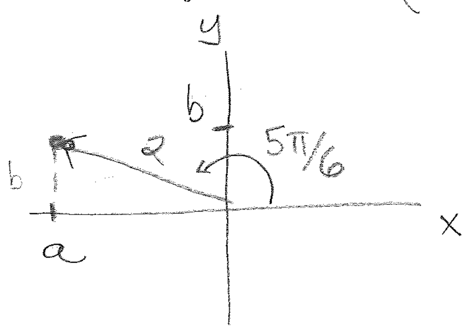
$|z| =$ $\sqrt{34}$

2 (c) Write the conjugate of z and graph it. Conjugate of z: $3-5i$

2 (d) Find the opposite of z and graph it. Opposite of z: $-3-5i$

2 8. Graph the complex number then write it in standard form, a + bi:

$2cis\frac{5\pi}{6}$ $= 2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

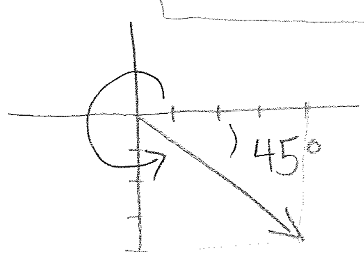


$$= 2\left(-\frac{\sqrt{3}}{2} + i\left(\frac{1}{2}\right)\right)$$

$$\boxed{= -\sqrt{3} + i}$$

4 9. Graph the complex number then write it in trig form, *rcisθ*

$4 - 4i$ $= \boxed{4\sqrt{2}\,cis\,315°}$ or $\boxed{4\sqrt{2}\,cis\,\frac{7\pi}{4}}$



$$|4-4i| = \sqrt{4^2+(-4)^2} = \sqrt{32} = 4\sqrt{2}$$

2 10. Convert the polar number $(6, \frac{7\pi}{6})$ to rectangular form.

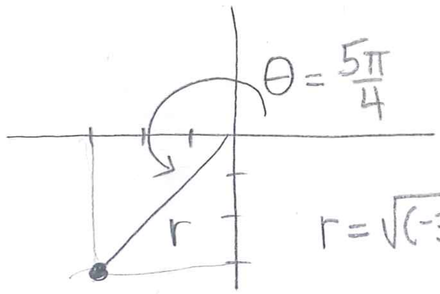
r θ

$$\boxed{(-3\sqrt{3}, -3)}$$

$$x = 6\cos\frac{7\pi}{6} = 6\left(-\frac{\sqrt{3}}{2}\right) = -3\sqrt{3}$$

$$y = 6\sin\frac{7\pi}{6} = 6\left(-\frac{1}{2}\right) = -3$$

8 11. Convert the rectangular number (-3, -3) to polar form



$(-3, 3) \sim (r, \theta)$

$(x, y) \sim \boxed{\left(3\sqrt{2}, \frac{5\pi}{4}\right)}$

or $\boxed{(3\sqrt{2}, 225°)}$

$r = \sqrt{(-3)^2 + (-3)^2} = \sqrt{18} = 3\sqrt{2}$

3 12. Convert the rectangular equation to a polar equation. Simplify the answer as much as possible.

$x^2 + y^2 = 4x$

$\frac{r^2}{r} = \frac{4r\cos\theta}{r}$

$\boxed{r = 4\cos\theta}$

3 13. Convert the polar equation to a rectangular equation (do not attempt to simplify!)

$r\sin\theta + r^2 = \frac{4}{1 - r\cos\theta}$

$\boxed{y + x^2 + y^2 = \frac{4}{1 - x}}$

Helpful formulas:

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c} \qquad c^2 = a^2 + b^2 - 2ab\cos(C)$$

$$\vec{v} \bullet \vec{w} = v_1 w_1 + v_2 w_2$$

$$\vec{v} \bullet \vec{w} = \|\vec{v}\| \|\vec{w}\| \cos\theta$$

Math 229: Test 4 (Take Home) **Name:**______________________________
(30 points)

*In-Class Test*__________*/70* *Take-Home Test*__________*/30*

- *This exam is due at the beginning of class on Tuesday, 12/12/2017 You may work with other people in the class but not with tutors, other instructors, etc.*
- *Be sure that all solutions are your own. If the work and the answer to the question don't match, the problem will receive zero credit.*
- *Your work should be neat, well-organized, and written in pencil so you can erase errors (Don't scribble out errors!)*

Do your work on **other paper** and attach this cover sheet to your work. Do NOT try to do the problems on this sheet (there isn't enough room).

1. (a) Draw an accurate sketch of the given sides and angle for Triangle ABC: $A = 40^O$, $b = 6$, $a = 5$. Arrange the triangle so that side c is horizontal (the base of the triangle).

   (b) Show, on the sketch above, why there are two solutions to this triangle.

   (c) Sketch each of the possible solution triangles and solve each completely (find all missing sides and angles). Show work!

2. For the following problems, you will use Euler's formula: $e^{i\theta} = \cos\theta + i\sin\theta$

   $e^{i\theta}$ is called "Euler's form" of a complex number
   $\cos\theta + i\sin\theta$ is called "trigonometric form"
   a + bi is called "standard form"

   (a) Demoivre's Theorem: DeMoivre's Theorem states that $(rcis\theta)^n = r^n cis(n\theta)$

   Use Euler's Formula to prove this (very short proof!). Begin on the left-side of the equation and use Euler's Formula and algebra to move to the right.

   (b) Use DeMoivre's Theorem to find $(2+2i)^6$

3. Go to http://audioundone.com/directional-responses-and-polar-diagrams-of-microphones and read the article about microphones.

   (a) What are the 4 types of microphones described in this article?

   (b) Find the equation mentioned in the article for the Cardiod Microphone. Graph this equation by hand. Construct a table of 13 points for (r, θ), with $0^O \le \theta \le 360^O$ using an increment of $\Delta\theta = 30^O$)

   (c) Was FIGURE 3.4 in the article an accurate graph of this equation?

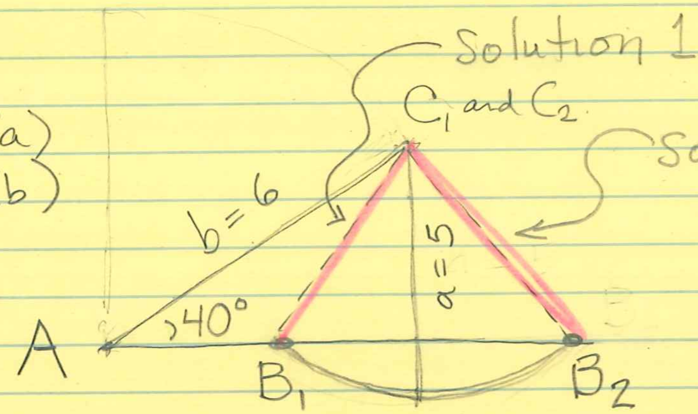
   (d) What is the correct equation for FIGURE 3.4?

   (e) Unlike what your instructor may have said in class (*sheepish look*), is the polar coordinate graph format (with the circles) actually used in real world applications?

Talk about sig figs!!

1.

(a)
(b)



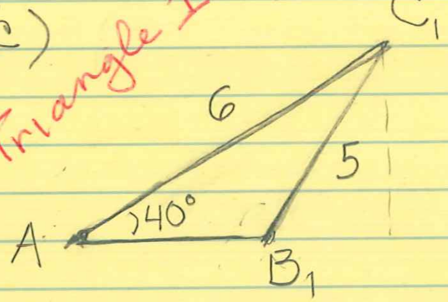
$$\frac{\sin B}{6} = \frac{\sin 40°}{5}$$

$$\sin B = \frac{6}{5}\sin 40°$$

$$\sin B = .7713$$

$B = 50.5°$ OR
$B = 180° - 50.5° = 129.5°$

(c) Triangle 1



$$\boxed{B_1 = 129.5°}$$

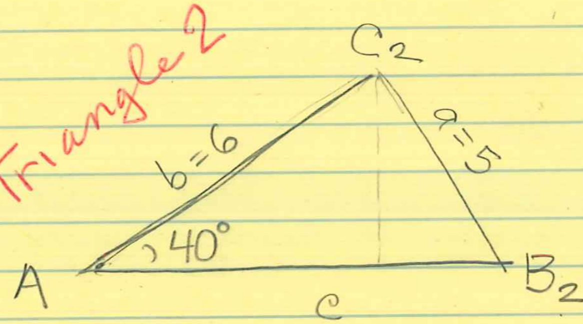
$$\boxed{C_1 = 10.5°}$$

$$\frac{\sin 10.5°}{c_1} = \frac{\sin 40°}{5}$$

$$c_1 = \frac{5\sin 10.5°}{\sin 40°}$$

$$\boxed{c_1 = 1.4}$$

Triangle 2



$$B_2 = 50.5°$$

$$C_2 = 89.5°$$

$$\frac{\sin 89.5°}{c_2} = \frac{\sin 40°}{5}$$

$$c_2 = \frac{5\sin 89.5°}{\sin 40°}$$

$$\boxed{c_2 = 7.8}$$

Triangle 1	Triangle 2
$a=5$	$a=5$
$b=6$	$b=6$
$c_1=1.4$	$c_2=7.8$
$A=40°$	$A=40°$
$B_1=129.5°$	$B_2=50.5°$
$C_1=10.5°$	$C_2=89.5°$

2. (a)

3 $(r\,\text{cis}\,\theta)^n$ — substitute: $\text{cis}\,\theta = e^{i\theta}$ Euler's Formula

$= (re^{i\theta})^n$ — distribute exponent n and multiply powers

$= r^n e^{i\theta n}$ — regroup

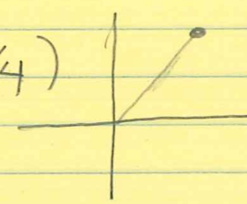
$= r^n e^{i(n\theta)}$ — Substitute: $e^{i(n\theta)} = \text{cis}(n\theta)$ Euler's Formula

$= r^n\,\text{cis}(n\theta)$

Q.E.D.

7 (b) $(2+2i)^6$

$2+2i$
$= 4\sqrt{2}\,\text{cis}(\pi/4)$



$|2+2i|$
$= \sqrt{2^2+2^2}$
$\sqrt{8}$
or $8^{\frac{1}{2}}$

$= \left(\sqrt{8}\,\text{cis}\,\pi/4\right)^6$

$= \left(8^{1/2}\right)^6 \text{cis}\left(6 \cdot \frac{\pi}{4}\right)$

$= 8^3\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$

$= 512\,(0 + i(-1))$

$= \boxed{-512i}$

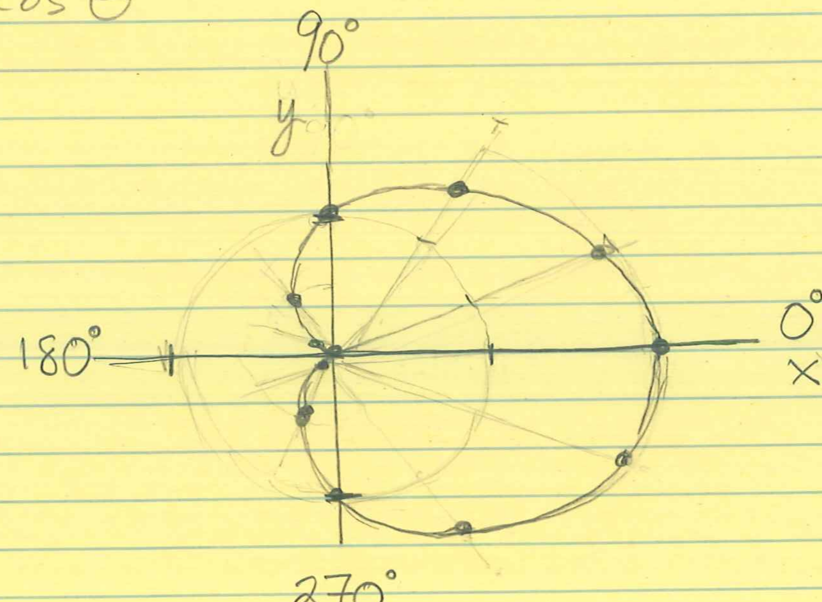
3 (a) The 4 types of microphones are

1. omnidirectional
2. Figure 8 = bidirectional
3. Cardiod = unidirectional
4. Hypercardiod

(b) Equation for cardiod: $1+\cos\theta$

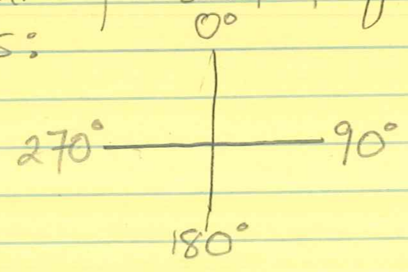
$r = 1+\cos\theta$

θ	r
0	2
30	1.9
60	1.5
90	1
120	.5
150	.1
180	0
210	.1
240	.5
270	1
300	1.5
330	1.9
360	2



(c) Initially I thought, no, the graph is not accurate since it appears to be rotated 90° ccw.

However on closer inspection (thank you, Riley!) the axes are presented quite differently as follows:

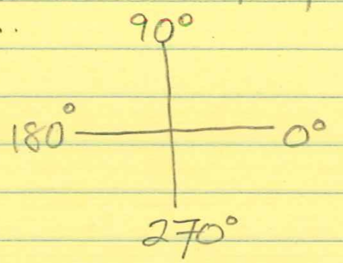


This suggests they are using clockwise as positive rotation!

3 (c) continued

sooo... based on the axes and rotation orientation, the graph depicted <u>IS</u> correct.

(d) IF we assume the axes as pictured are oriented per our normal way, i.e., ..



then the correct equation is

$$r = 1 + \sin\theta$$

(e) yes – however the change in which rotational direction is positive goes to show that coordinate systems can be manipulated and morphed to fit particular applications, (BUT in math, the underlying reference system is the xy-coordinate system!)

[illegible] are oriented for our normal way [illegible]

[illegible] the correct [illegible]

[illegible] is

[illegible] = 1+ sin θ

(c) [illegible] the change of rotational direction is positive goes to show that coordinate systems can be manipulated and morphed to fit particular applications