Math 229: Test 4
(Wright - Fall 2018)

Name: KEY

Please do your work in a well-organized manner. Credit is based on the amount of <u>correct</u> work shown, not just on the final answer. Use proper notation. Only *scientific calculators* are allowed on the exam.

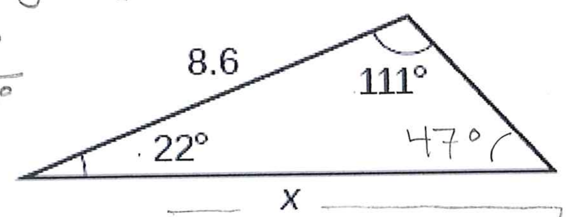
Some helpful formulas are located on the last page!

The following problems refer to a triangle ABC which has angles and/or sides as given. Solve for the indicated side or angle.

1. (8 pts) Solve for side x, as shown, in the triangle.

Missing angle

$$180^\circ - 133^\circ = 47^\circ$$ 2pts



$$\frac{\sin(111^\circ)}{x} = \frac{\sin(47^\circ)}{8.6}$$ 3pts

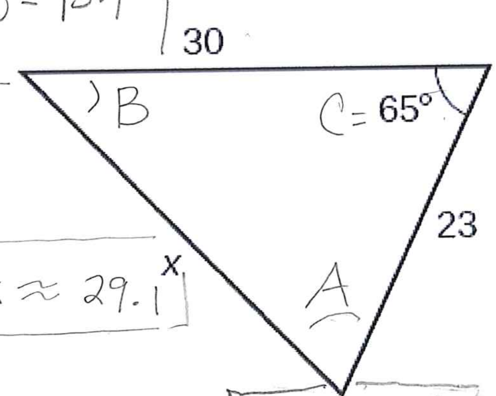
$$\frac{8.6\sin(111^\circ)}{\sin(47^\circ)} = \frac{x\sin(47^\circ)}{\sin(47^\circ)}$$

$$x = \frac{8.6\sin(111^\circ)}{\sin(47^\circ)} \approx \boxed{10.98 \text{ or } 11 \text{ (2 sig figs)}}$$

3pts $\boxed{x \approx 10.98 \approx 11}$

radian error $x = 8.7$

2. (14 pts) Given the triangle as shown,
(a) Solve for side x.



$\boxed{B = 45.9^\circ}$

$\boxed{x \approx 29.1}$

$\boxed{A = 69.1^\circ}$

3pts $x^2 = 30^2 + 23^2 - 2(30)(23)\cos(65^\circ)$

2pts $x^2 \approx 845.787$

2pts $\boxed{x \approx 29.1}$

(b) Find the other angles in the triangle.

2pts $$\frac{\sin 65^\circ}{29.1} = \frac{\sin A}{30}$$

$$\frac{29.1\sin A}{29.1} = \frac{30\sin 65^\circ}{29.1}$$

$$\sin A = \frac{30\sin 65^\circ}{29.1} \approx .9343$$

3pts $A = \sin^{-1}(.9343)$

$\boxed{A = 69.1^\circ}$

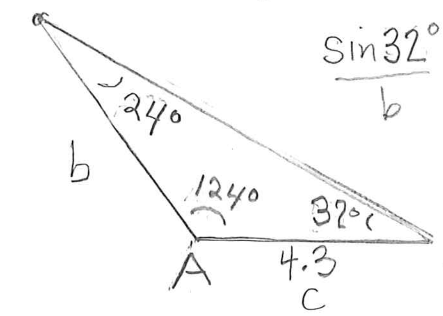
2pts $B = 180^\circ - (65^\circ + 69.1^\circ)$

$\boxed{B = 45.9^\circ}$

3. (8 pts) A pilot is flying over a straight highway. She determines the angles of depression to two mileposts, 4.3 km apart, to be 32° and 56°, as shown in the picture.

4 (a) Find the distance from the plane to point A. Round your answer to the nearest tenth of a kilometer.

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

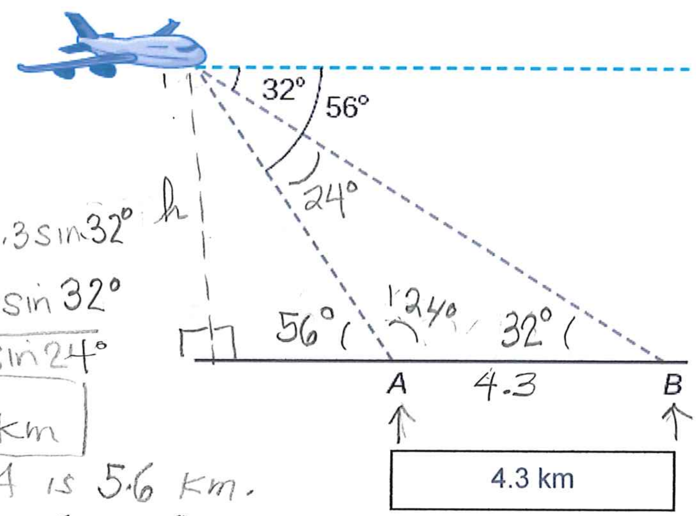


$$\frac{\sin 32^\circ}{b} = \frac{\sin 24^\circ}{4.3}$$

$$b \cdot \sin 24^\circ = 4.3 \sin 32^\circ$$

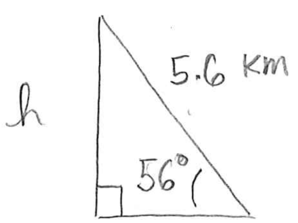
$$b = \frac{4.3 \sin 32^\circ}{\sin 24^\circ}$$

$$\boxed{b = 5.6 \text{ km}}$$



The distance from the plane to A is 5.6 km.

4 (b) Find the elevation of the plane (height directly above the ground).



$$\frac{h}{5.6} = \sin(56^\circ)$$

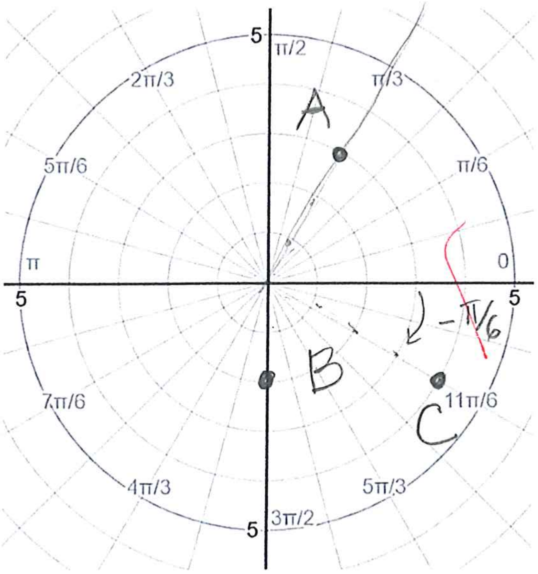
$$h = 5.6 \sin(56^\circ)$$

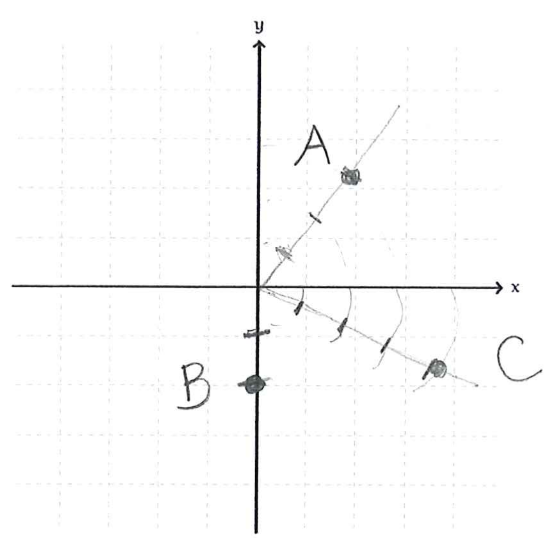
$$\boxed{h = 4.6 \text{ km}}$$

The elevation of the plane is 4.6 km

4. (6 pts) Plot the polar points. You may either use the polar grid provided or plot them on the xy-coordinate system,,,your choice!. LABEL the points.

Point A: $\left(3, \frac{\pi}{3}\right)$ Point B: $\left(-2, \frac{\pi}{2}\right)$ Point C: $\left(4, -\frac{\pi}{6}\right)$





Use:
$x = r\cos\theta$
$y = r\sin\theta$

5. (6 pts) Convert the point $\left(5, \frac{-2\pi}{3}\right)$ to rectangular form. Show work for credit!

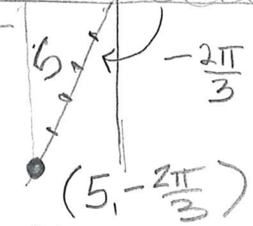
(r, θ)

$x = r\cos\theta$
$= 5\cos\left(-\frac{2\pi}{3}\right)$
$= 5\left(-\frac{1}{2}\right)$
$= -\frac{5}{2}$

$y = r\sin\theta$
$= 5\sin\left(-\frac{2\pi}{3}\right)$
$= 5\left(-\frac{\sqrt{3}}{2}\right)$
$= -\frac{5\sqrt{3}}{2}$

(x, y)
$\boxed{= \left(-\frac{5}{2}, -\frac{5\sqrt{3}}{2}\right)}$
$\approx (-2.5, -4.3)$

$x^2 + y^2 = r^2$
$\tan\theta = \frac{y}{x}$



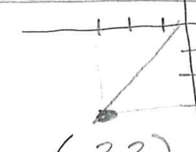
6. (6 pts) Convert the rectangular number (-3, -3) to polar form. Show work for credit!

(x, y)

$r^2 = x^2 + y^2$
$r^2 = (-3)^2 + (-3)^2$
$= 18$
$r = \sqrt{18}$ or $3\sqrt{2}$

$\theta_{Ref} = \tan^{-1}\left|\frac{-3}{-3}\right|$
$\theta_{Ref} = \frac{\pi}{4}$
$\theta = \frac{5\pi}{4}$ (QIII)

(r, θ)
$\boxed{\left(3\sqrt{2}, \frac{5\pi}{4}\right)}$
$\approx (4.2, 3.9)$



$\frac{\pi}{4}$ instead of $\frac{5\pi}{4}$

7. (6 pts) Convert the rectangular equation to a polar equation. Simplify the answer as much as possible.

-1

$x^2 + y^2 = 4x$

$x^2 + y^2 = r^2$
$x = r\cos\theta$

$r^2 = 4r\cos\theta$

$\boxed{r = 4\cos\theta}$

8. (8 pts) Convert the polar equation to a rectangular equation. (Hint on (b): Clear the fraction!!!)

(a) $r = 7\cos\theta$

$r^2 = 7r\cos\theta$

$\boxed{x^2 + y^2 = 7x}$

(b) $r = \dfrac{-2}{4\cos\theta + \sin\theta}$

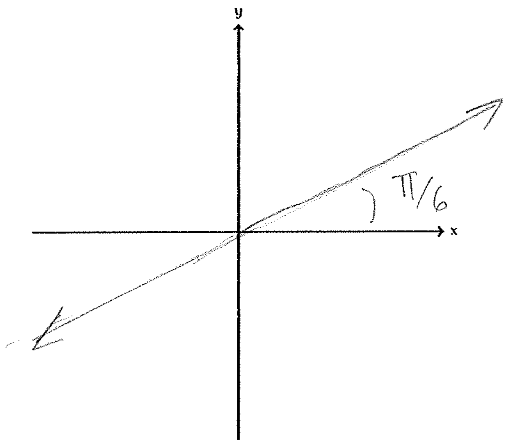
$r(4\cos\theta + \sin\theta) = -2$

$4r\cos\theta + r\sin\theta = -2$

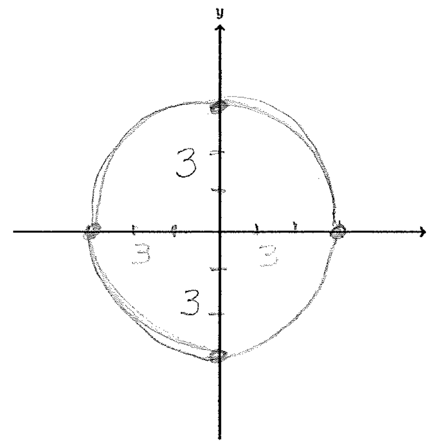
$\boxed{4x + y = -2}$

9. (6 pts) Graph each of the polar equations. No work is necessary and you do not need to convert to rectangular coordinates!

(a) $\theta = \frac{\pi}{6}$



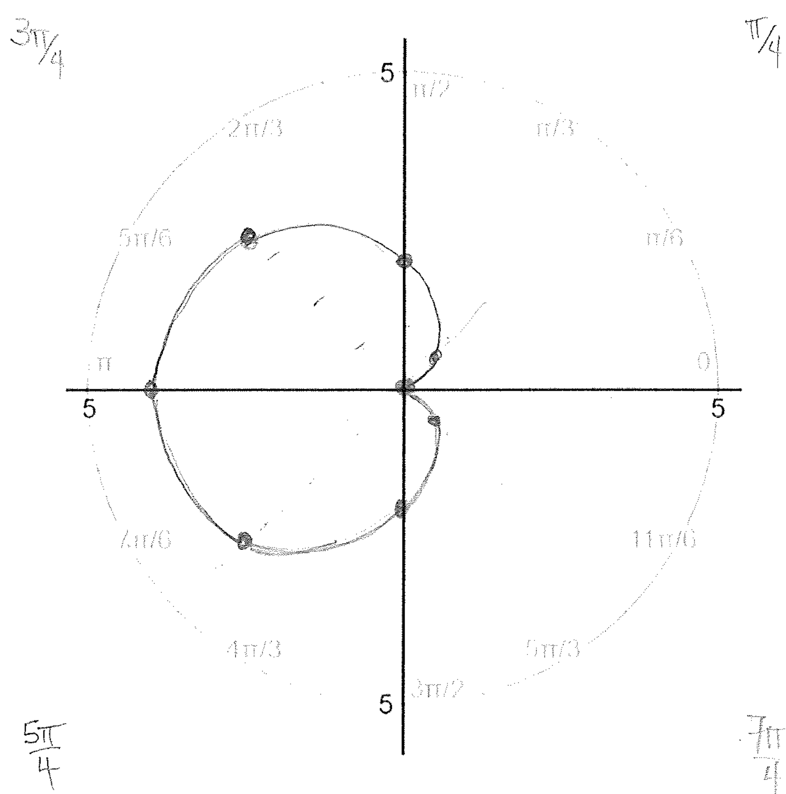
(b) $r = 3$



10. (12 pts) Graph the polar equation $r = 2 - 2\cos\theta$. For full credit, you must include a table with 8 points, using $\frac{\pi}{4}$ as your increment.

Table:

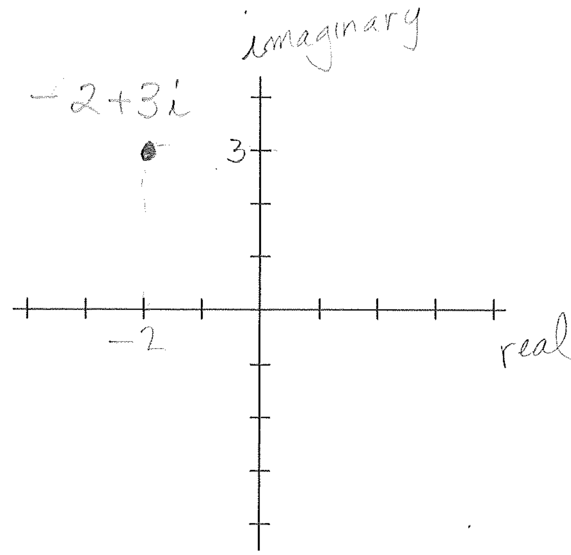
θ	$r = 2 - 2\cos\theta$
0	0
$\frac{\pi}{4}$	$.6$
$\frac{\pi}{2}$	2
$\frac{3\pi}{4}$	3.4
π	4
$\frac{5\pi}{4}$	3.4
$\frac{3\pi}{2}$	2
$\frac{7\pi}{4}$	$.6$
2π	0



11. (8 pts) Given the complex number $z = -2 + 3i$

(a) Graph z on the Complex Plane.

(b) Find the absolute value of z. Leave in exact terms.



$$|z| = \sqrt{a^2 + b^2}$$
$$= \sqrt{(-2)^2 + (3)^2}$$
$$= \sqrt{13}$$

$\boxed{|z| = \underline{\sqrt{13}}}$

12. (6 pts) Convert the complex number from polar to rectangular form. Show work for credit!
Round your answer to one decimal place. Note: $cis\,\theta = \cos\theta + i\sin\theta$
$z = 3cis(40^\circ)$,

$$z = 3(\cos 40^\circ + i\sin 40^\circ)$$
$$= 3\cos 40^\circ + 3\sin 40^\circ i$$
$$\boxed{= 2.3 + 1.9i}$$

13. (6 pts) Write the complex number $z = 1 + \sqrt{3}i$ in polar form. Show work for credit and express your answer in exact terms.

$z = a + bi = |z|\,cis\,\theta$

$\theta_{Ref} = \tan^{-1}\frac{\sqrt{3}}{1} = \frac{\pi}{3}$

$$|z| = \sqrt{(1)^2 + (\sqrt{3})^2}$$
$$= \sqrt{4}$$
$$= 2$$

$$\boxed{z = 2cis\left(\frac{\pi}{3}\right)}$$

Helpful formulas:

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c} \qquad c^2 = a^2 + b^2 - 2ab\cos(C)$$

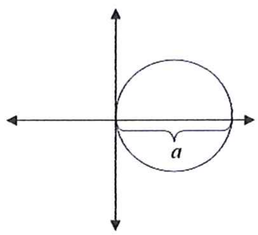
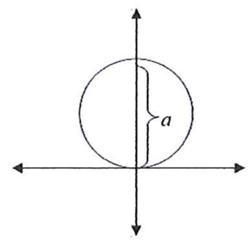
$$|z| = \sqrt{a^2 + b^2} \qquad z = a + bi = |z|cis\theta, \quad \tan\theta = \tfrac{b}{a}$$

Math 229: Graphs of Common Polar Equations Summary (for Test 4)

<u>**Lines** in Polar Coordinates:</u>

Lines through the Origin:	Vertical Lines	Horizontal Lines
Rectangular: $y = mx$ Polar: $\theta = \theta_O,\ m = \tan(\theta_O)$	Rectangular: $x = a$ Polar: $r = a\sec(\theta)$	Rectangular: $y = b$ Polar: $r = a\csc(\theta)$

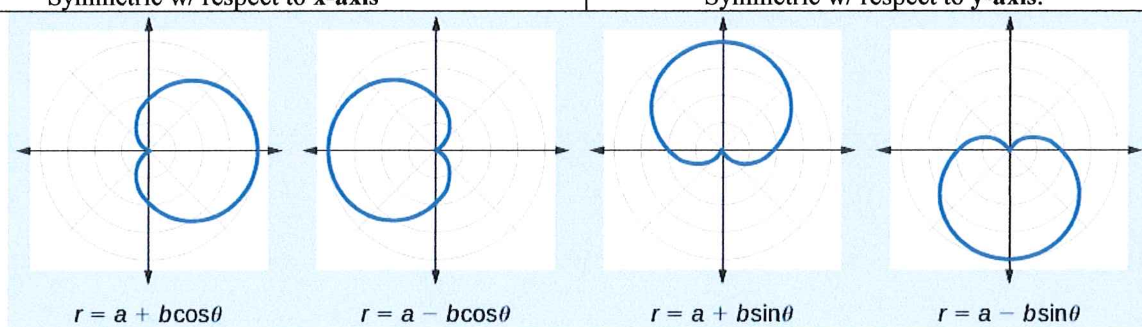
<u>**Circles** in Polar Coordinates:</u>

$r = a\cos\theta$ 	$r = a\sin\theta$ 

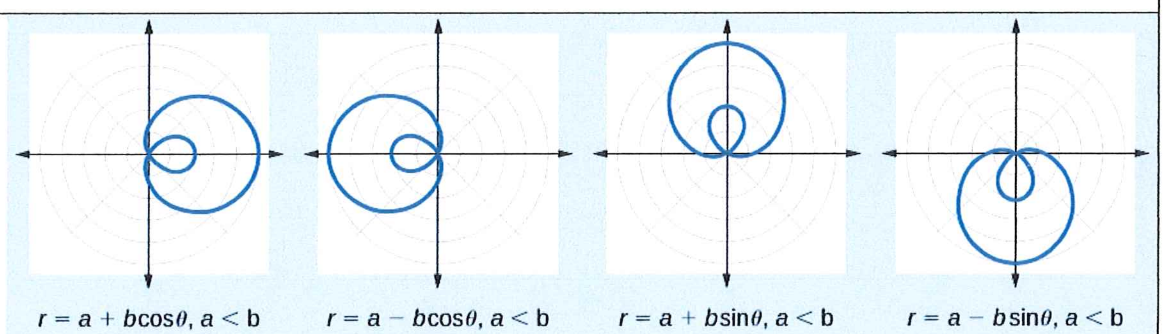
<u>**Limaçons**:</u>

	$r = a + b\cos\theta$ Symmetric w/ respect to **x-axis**	$r = a + b\sin\theta$ Symmetric w/ respect to **y-axis.**

If $\lvert a\rvert = \lvert b\rvert$, creates a heart-shaped **cardiod** which has a "cusp".



If $\lvert a\rvert < \lvert b\rvert$, creates an **inner loop**:



If $\lvert a\rvert > \lvert b\rvert$, creates a curves with **neither** cusp **nor** inner loop