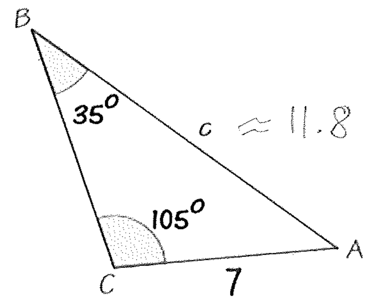
Math 229: Test 4 **Name:** KEY
(Wright – Spring, 2019)

Please do your work in a well-organized manner. **Credit is based on the amount of correct work shown, not just on the final answer**. Use proper notation. Only *scientific calculators* are allowed on the exam.

Some helpful formulas are located on the last page!

The following problems refer to a triangle ABC which has angles and/or sides as given. Solve for the indicated side or angle.

1. (8 pts) Solve for side c, as shown, in the triangle.



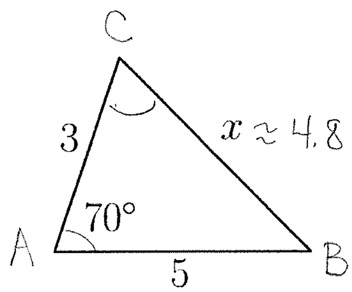
$$\frac{\sin(105°)}{c} = \frac{\sin(35°)}{7}$$

$$c = \frac{7\sin(105°)}{\sin(35°)} \approx 11.8$$

$$\boxed{c \approx 11.8}$$

2. (12 pts) Given the triangle as shown,

6 (a) Solve for side x.



$$x^2 = 3^2 + 5^2 - 2(3)(5)\cos(70°)$$

$$x^2 = 23.739\ldots$$

$$x = \sqrt{23.739\ldots} \approx 4.8723\ldots$$

$$\boxed{x \approx 4.9}$$

4 (b) Find the other two angles in the triangle.

$$\frac{\sin C}{5} = \frac{\sin 70°}{4.8723}$$

$$\sin C = \frac{5\sin(70°)}{4.8723}$$

$$\sin C \approx .9643$$

$$C = \sin^{-1}(.9643\ldots)$$

$$\boxed{C = 74.6°}$$

acceptable 74.9° (best answer!)
73.5° (too much rounding!)

Note: Answers varied a LOT based on rounding!

2. $B = 180° - A - C$

$$= 180° - 70° - 74.9°$$

$$\boxed{B = 35.4°}$$

3. (4 pts) (a) Use the Law of Sines to show that no triangle exists for which $A = 60^{\circ}$, a = 1 in, and b = 3 in.

$$\frac{\sin 60^{\circ}}{1} = \frac{\sin(B)}{3}$$

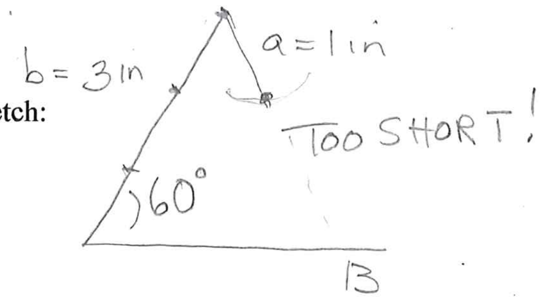
$$\sin(B) = 3\sin(60^{\circ}) = 2.598\cdots$$

$$B = \sin^{-1}(2.598\cdots) \Rightarrow \text{undefined!}$$

Such a B does not exist since $\sin(B) > 1$ which is not possible.

(b) Extra Credit (2 points) Make a somewhat accurate sketch of the given sides and angle to illustrate why Triangle ABC doesn't exist.

Sketch:



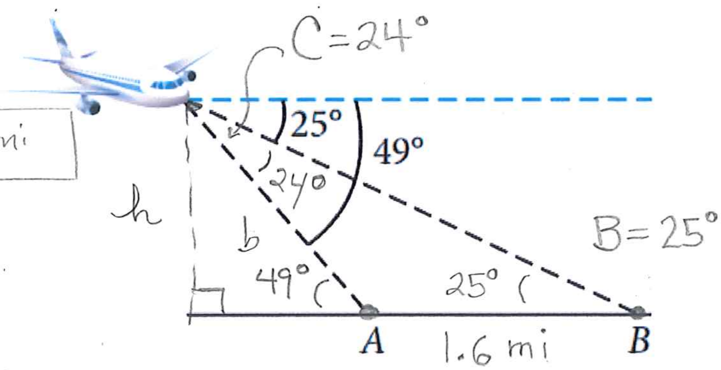
4. (8 pts) A pilot is flying over a straight highway. She determines the angles of depression to two points, A and B, to be 25° and 49°, as shown in the picture. The distance between A and B is 1.6 miles.

4 (a) Find the distance from the plane to point A. Round your answer to the nearest tenth of a mile.

$$\frac{\sin 25^{\circ}}{b} = \frac{\sin 24^{\circ}}{1.6}$$

$$b = \frac{1.6\sin(25^{\circ})}{\sin(24^{\circ})} \approx \boxed{1.66\cdots \text{ mi}}$$

The distance from the plane to point A is approximately $\boxed{1.7 \text{ mi}}$



4 (b) Find the elevation of the plane (height directly above the ground).

$$\frac{h}{b} = \sin(49^{\circ})$$

$$h = b \cdot \sin(49^{\circ}) \quad (b = \text{distance above})$$

$$h \doteq \boxed{1.25\cdots \text{ mi}}$$

The elevation of the plane is approx. $\boxed{1.3 \text{ mi}}$

5. (6 pts) (a) Plot the Cartesian point $(4, -4)$.

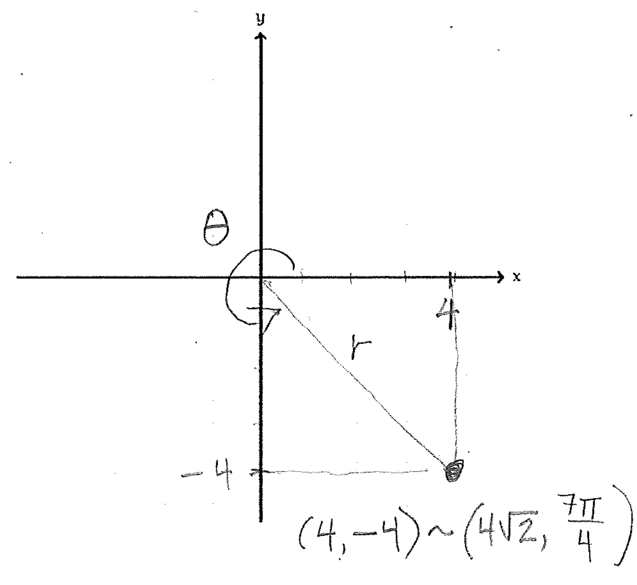
0

3 (b) What is r for the polar coordinates of this point? Show work and leave in exact form.

$$r = \sqrt{x^2 + y^2}$$
$$r = \sqrt{(4)^2 + (-4)^2}$$
$$= \sqrt{32}$$
$$= 4\sqrt{2}$$

2 (c) What is θ? $0 \le \theta \le 2\pi$ $\quad \theta = \frac{7\pi}{4}$

1 (d) Write the point in Polar Coordinates: $\left(4\sqrt{2}, \frac{7\pi}{4}\right)$



6. (10 pts) (a) Plot and label (A,B,C) the polar points.

Point A: $\left(4, \frac{5\pi}{6}\right)$

Point B: $\left(-3, \pi\right)$

Point C: $\left(6, -\frac{\pi}{4}\right)$

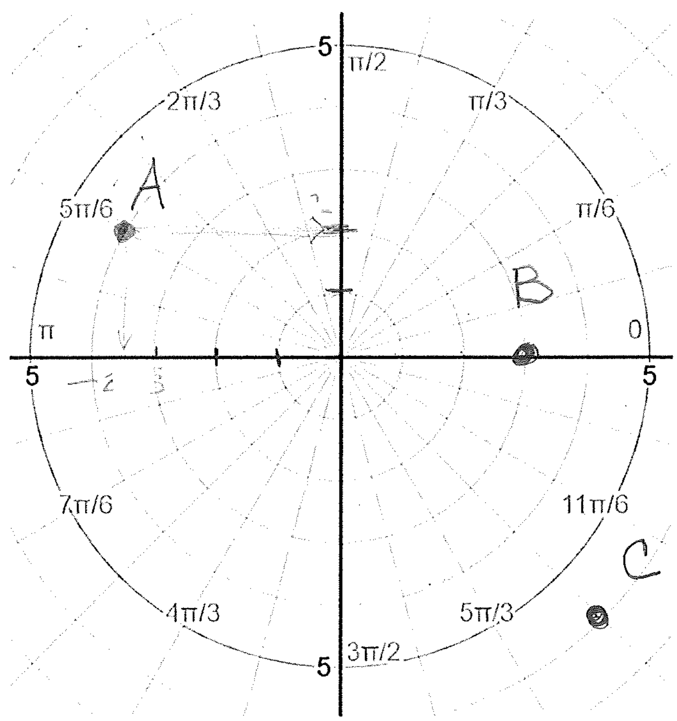
(b) Convert Point A $\left(4, \frac{5\pi}{6}\right)$ to Cartesian Coordinates.

Show work and leave in exact form.

$$x = r\cos\theta$$
$$= 4\cos\left(\frac{5\pi}{6}\right)$$
$$= 4\left(-\frac{\sqrt{3}}{2}\right) = -2\sqrt{3}$$

$$y = r\sin\theta$$
$$= 4\sin\left(\frac{5\pi}{6}\right)$$
$$= 4\left(\frac{1}{2}\right) = 2$$

$$\boxed{A = (-2\sqrt{3}, 2)}$$
$$\approx (-3.5, 2)$$



Note: checks out on graph!

7. (6 pts) Convert each rectangular equation to a polar equation. Solve for r in each case.

(a) $x^2 + y^2 = 4$

$r^2 = 4$

$\boxed{r = 2}$

(b) $y = 7$

$r\sin\theta = 7$

$\boxed{r = \frac{7}{\sin\theta}}$ (ok)

OR

$\boxed{r = 7\csc\theta}$

8. (6 pts) Convert each polar equation to a rectangular equation.

(a) $r = 6\sin\theta$

$r = 6\frac{y}{r}$

$r^2 = 6y$

$\boxed{x^2 + y^2 = 6y}$

(b) $\left(r = \frac{4}{\cos\theta}\right) \cdot \cos\theta$

$r \cdot \cos\theta = \frac{4}{\cancel{\cos\theta}} \cdot \cancel{\cos\theta}$

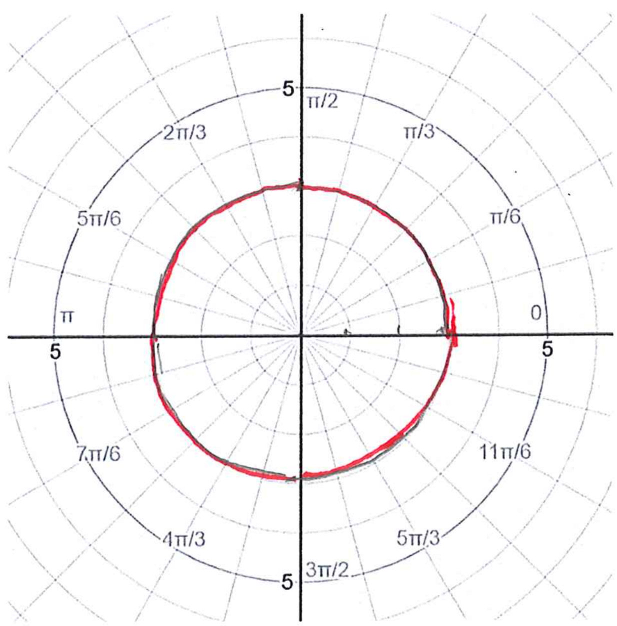
$\boxed{x = 4}$

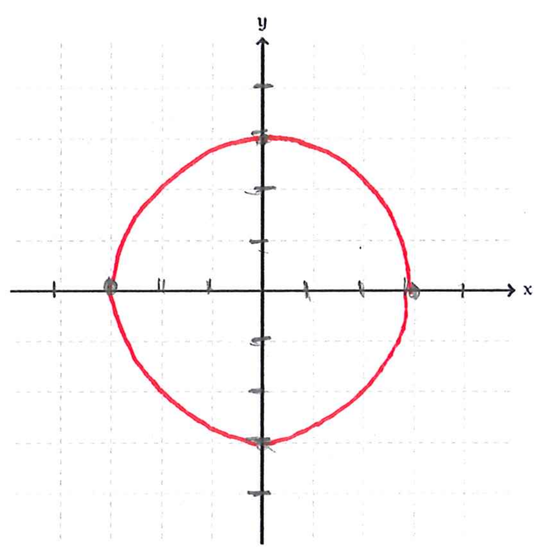
FYI: Convert $r = 3$

$\sqrt{x^2 + y^2} = 3$

$x^2 + y^2 = 9$

9. (4 pts) Graph the polar equation $r = 3$ on both coordinate grids.
No work is necssary and you do not need to convert to rectangular coordinates!



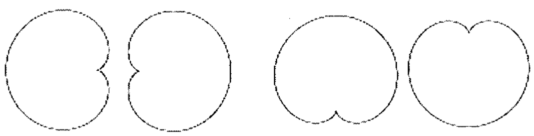


10. (8 pts) Graph the polar equation $r = 3 - 3\cos\theta$.

<u>For full credit</u>, you must include a table with <u>8 points</u>, using $\frac{\pi}{4}$ as your increment.

For reference:

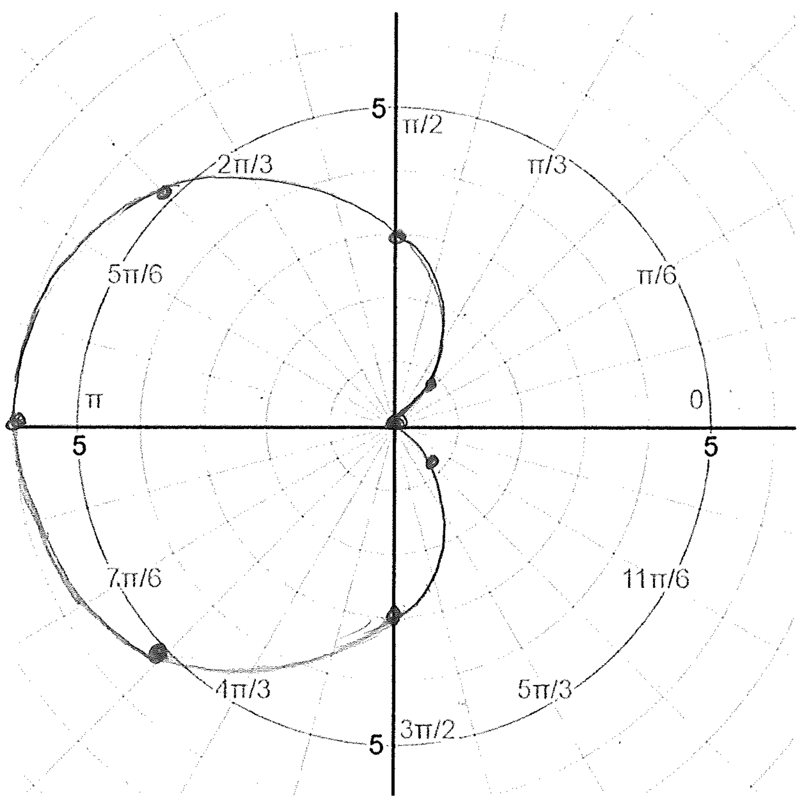
Horizontal Cardioids | Vertical Cardioids



$r = a \pm a\cos\theta$ | $r = a \pm a\sin\theta$

Table:

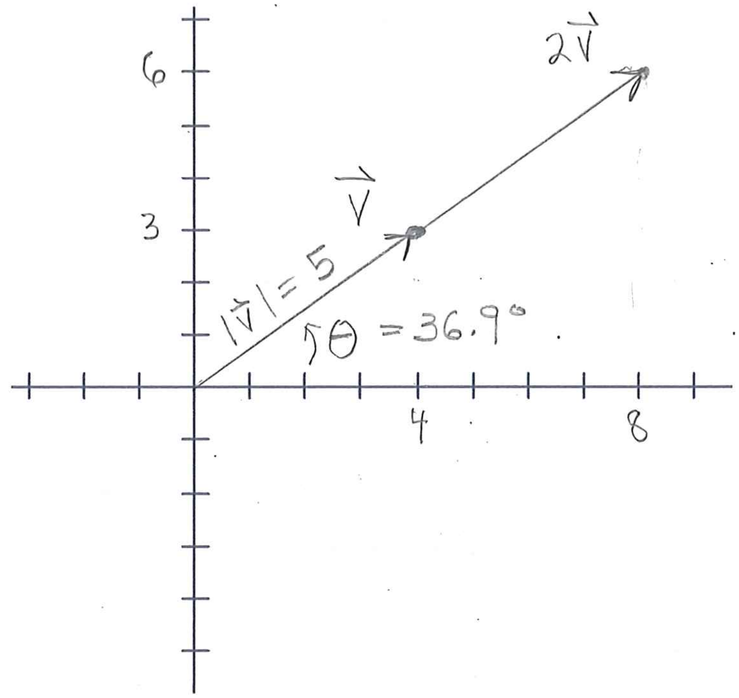
θ	r
0	$3-3\cos(0)=0$
$\frac{\pi}{4}$	$3-3\cos(\pi/4)=.9$
$\frac{\pi}{2}$	$3-3\cos(\frac{\pi}{2})=3$
$\frac{3\pi}{4}$	$3-3\cos(\frac{3\pi}{4})=5.1$
π	$3-3\cos(\pi)=6$
$\frac{5\pi}{4}$	$3-3\cos(\frac{5\pi}{4})=5.1$
$\frac{3\pi}{2}$	$3-3\cos(\frac{3\pi}{2})=3$
$\frac{7\pi}{4}$	$3-3\cos(\frac{7\pi}{4})=.9$
2π	$3-3\cos(2\pi)=0$



11. (12 pts) For vector $\vec{v} = 4\hat{i} + 3\hat{j} = \langle 4,3 \rangle$

3 (a) Graph $\vec{v}$ and $2\vec{v}$

$\langle a, b \rangle$



3 (b) Find the magnitude of $\vec{v}$. Show work!

$$|\vec{v}| = \sqrt{a^2 + b^2}$$
$$= \sqrt{(4)^2 + (3)^2}$$
$$= \sqrt{25}$$
$$= 5$$

$$\boxed{|\vec{v}| = 5}$$

3 (c) Find the direction of $\vec{v}$ in degrees (standard angle).
Round the angle to one decimal place.

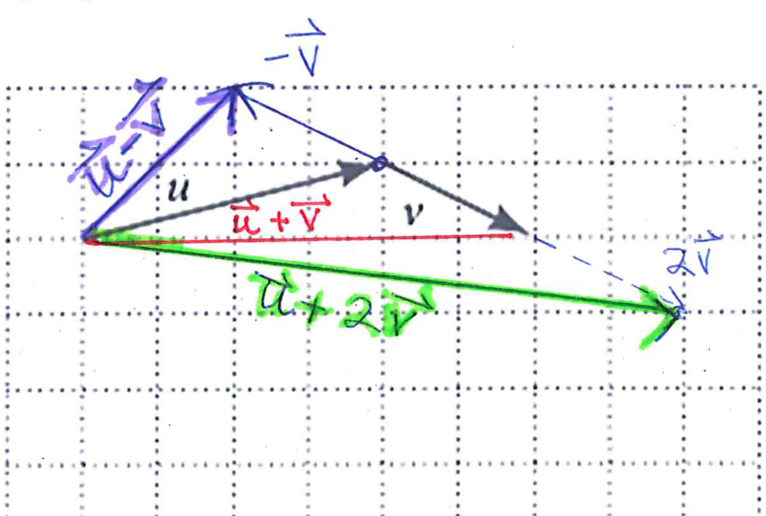
$$\tan\theta = \frac{b}{a} = \frac{3}{4}$$
$$\theta = \tan^{-1}\left(\frac{3}{4}\right)$$
$$\boxed{\theta \approx 36.9°}$$

3 (d) Find a unit vector, $\vec{u}$, in the same direction as $\vec{v}$. Illustrate $\vec{u}$ on the graph of $\vec{v}$

$$\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \frac{1}{5}\langle 4, 3 \rangle = \boxed{\left\langle \frac{4}{5}, \frac{3}{5} \right\rangle}$$

Check: $|\vec{u}| = 1$? $\quad |\vec{u}| = \sqrt{\frac{16}{25} + \frac{9}{25}} = \sqrt{\frac{25}{25}} = 1$ ✓

12. (6 pts) Graph $\vec{u} + \vec{v}$, $\vec{u} - \vec{v}$, and $\vec{u} + 2\vec{v}$ on the given grid. Label each!



13. (6 pts) Use the vectors $\vec{v} = \langle 3,-2\rangle$ and $\vec{w} = \langle 1,5\rangle$ to find each of the following:

(a) $-5\vec{v} + 6\vec{w}$

$= -5\langle 3,-2\rangle + 6\langle 1,5\rangle$

$= \langle -15, +10\rangle + \langle 6,30\rangle = \boxed{\langle -9,40\rangle}$

(b) $\vec{v} \bullet \vec{w}$

$= \langle 3,-2\rangle \bullet \langle 1,5\rangle$

$= 3\cdot 1 + (-2)(5)$

$= 3 - 10 = \boxed{-7}$

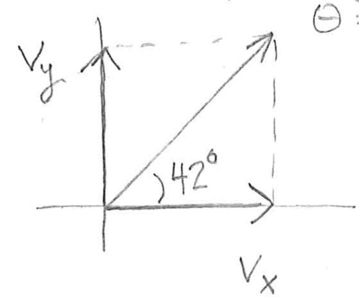
14. (4 pts) Suppose Bella throws a baseball with an initial velocity of 55 feet per second at an angle of 42°. Find the horizontal and vertical components of the velocity vector of the ball. Round to one decimal place.

$\vec{V} = 55$ ft/s

$\theta = 42°$

$|\vec{V}| = 55$ ft/sec

$\theta = 42°$



$V_x = |\vec{v}|\cos\theta$

$= 55\cos(42°)$

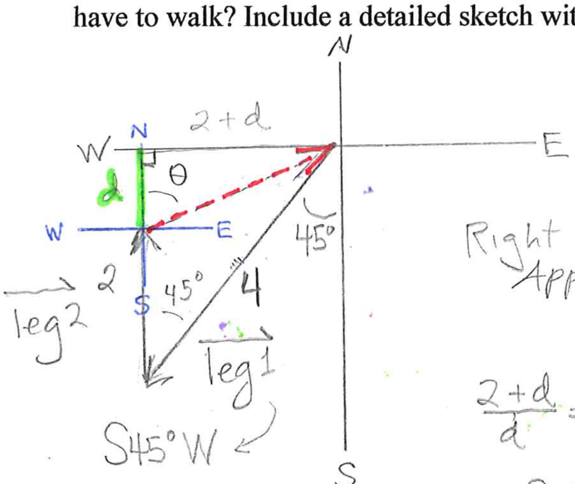
$V_y = |\vec{v}|\sin\theta$

$= 55\sin(42°)$

$\boxed{V_x = 40.9 \text{ ft/sec horizontal}}$ $\boxed{V_y = 36.8 \text{ ft/sec vertical}}$

Extra credit (3 pts):

Carlos is backpacking in the Sierras. From his starting point, he hiked 4 miles in the southwest direction. He then turned and hiked 2 miles due north. If he wants to hike directly back to his starting point, in what direction will he have to walk? Include a detailed sketch with the solution.



Carlos should walk in the N 73.7° E direction

$\vec{V}$ = return vector

Direction: N θ E

There are MANY strategies for finding θ

Right Triangles Approach: $\frac{2+d}{4} = \sin 45°$

$\Rightarrow 2 + d = 4\sin(45°)$

$d = 4\sin(45°) - 2 \approx .828\ldots$

$\frac{2+d}{d} = \tan\theta$

$\Rightarrow \theta = \tan^{-1}\left(\frac{2+d}{d}\right) = 73.7°$

You could find side; then the obtuse angle to get direction

Helpful formulas: $\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$ $c^2 = a^2 + b^2 - 2ab\cos(C)$