Inductive and Deductive Reasoning

Inductive reasoning means drawing generalizations out of specific observations. Read the following Wikipedia entry, which has a useful description and examples of this type of reasoning. http://en.wikipedia.org/wiki/Inductive_reasoning

Deductive reasoning involves drawing a specific conclusion based on a set of premises which are assumed to be true. The formal system of reasoning called "symbolic logic" is based on deduction, but deductive reasoning is applied any time you reason from a general law to a specific conclusion. Take a look at the Wikipedia entry on deductive reasoning: http://en.wikipedia.org/wiki/Deductive_reasoning

Examples: Determine which type of reasoning is used in each of the following descriptions:

1) You went for a run on a hot day and got a headache. Your headache went away after you drank a quart of water. Your friend who also runs has noticed the same thing; i.e., headaches go away after drinking a lot of water. You conclude that headaches are caused by dehydration.

2) Electrocardiograms (ECG's) show a bump called a "Q-wave" when a person has had a heart attack. Mike's dad went in for a routine physical in which the doctor did an ECG and found a Q-wave. She informed him that he had had a heart attack some time in the past.

Answers: 1) This is inductive reasoning. You went from specific observations (your headache, and your friend's, going away after drinking water) to a generalization (headaches are caused by dehydration).

2) This is deductive reasoning. The accepted general "law" about ECG's is that the presence of a Q-wave indicates a person has had a heart attack. The doctor reasoned from that general law to the specific conclusion that Mike's dad had had a heart attack.

Let's see how the more formal structure of symbolic logic would look in Example 2. We could write the example as a logical "argument"

Premises: $\begin{cases} \text{All people with a Q-wave (on an ECG) have had heart attacks.} \\ \text{Mike's dad has a Q-wave.} \end{cases}$

Conclusion: $\therefore$ Mike's dad has had a heart attack

A logical argument consists of a set of premises which are assumed to be true and a conclusion. If the conclusion MUST follow from the premises, then the argument is valid; otherwise, it's considered invalid. If the conclusion might be true, but isn't guaranteed and literally FORCED to be true by the premises, then the argument is still considered invalid. This might go against your intuition, but the word "valid" in this context has a very strict meaning.

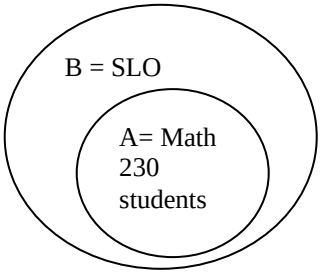
Euler Circles and Diagrams

Euler diagrams using Euler circles can help with determining whether a given argument is valid or not. First, a given statement is assigned a letter to represent it, then the circles are arranged as follows:

Example 1: Let A = Students in Math 230 and B = people who live in SLO.

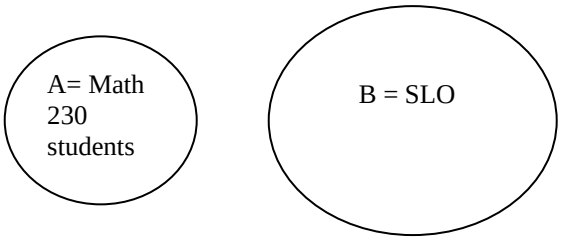
The statement "All Math 230 students live in SLO" would translate into "All A is B". The Euler diagram for this statement would be

All A is B



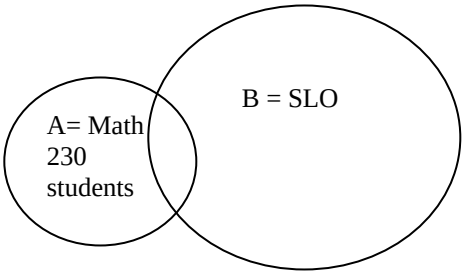
The statement "No Math 230 students live in SLO" would translate "No A is B". The Euler diagram for this is

No A is B



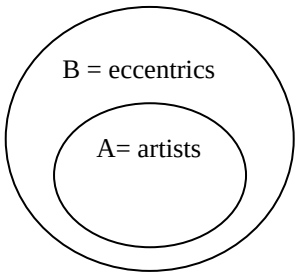
The statement "Some Math 230 students live in SLO would translate as "Some A is B". Again, here is the Euler diagram

Some A is B



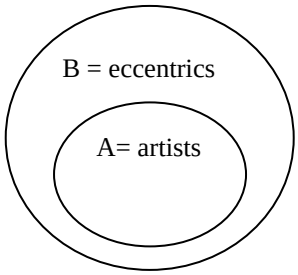
Example 2: Use an Euler diagram to determine if the following argument is valid:

All artists are eccentric.
Misa is eccentric.
$\therefore$ Misa is an artist.

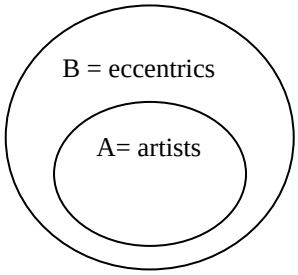
All A are B 	1. Diagram the first premise. A = artists, B = eccentrics. 2. Determine where the second premise falls within the circle(s). If possible, include the second premise in your diagram. 3. Is the conclusion FORCED to be true by where that second premise is located? If yes, then the argument is "valid". If the conclusion is NOT forced to be true (maybe, maybe not…) then the argument is "invalid". In this case, since Misa is just eccentric, she may fall inside OR outside the artist circle. She isn't FORCED to be in the artist circle, only in the eccentric circle, so this argument is INVALID.

Example 3: Question: Using the same statements, can you give an example of a valid argument?

Answer: Yes!

 All artists are eccentric. Misa is an artist. $\therefore$ Misa is eccentric.	No matter where we put Misa in the artist circle, she'll still be in the eccentric circle as well, so we can indeed conclude that Misa is eccentric! So this argument is VALID.

Example 4: What about negation? For instance, would the following argument be valid?



All artists are eccentric.
Cody is NOT an artist.
∴ Cody is NOT eccentric.

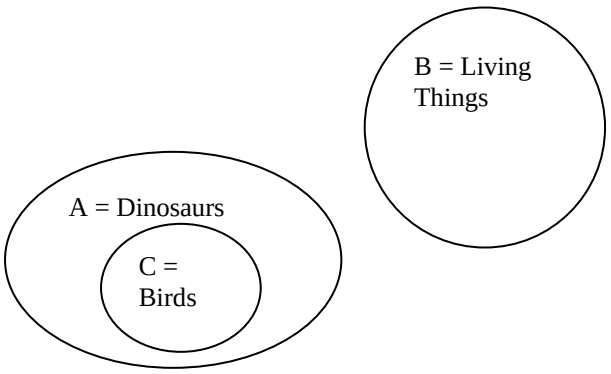
We can use the same Euler diagram, but when we look where to place Cody, since he's NOT an artist, he has to be somewhere <u>outside the artist circle</u>.

That means he could still be in the eccentric circle but he also could be outside of it.

Maybe he's eccentric, maybe he's not. We don't know so the argument is INVALID

Example 5: How would you analyze the validity of this argument:

No dinosaurs are alive.
All birds are dinosaurs.
∴ No birds are alive



In this problem we can use 3 Euler circles, since "birds" is an entire group (set) of objects, as opposed to Cody and Misa in the examples above who are just individuals.

Since the Birds are completely contained in the Dinosaur circle, none of the birds can be in the Living Things circle, so the conclusion is inescapably true; hence this argument is VALID.

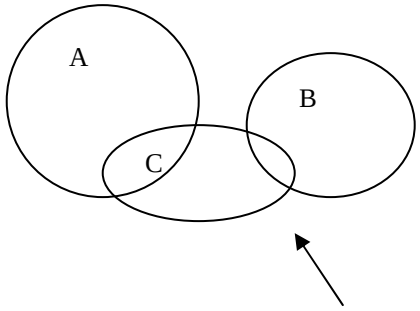
This argument isn't sound, however, since the second premise isn't true (birds may have evolved from dinosaurs but they're birds, not dinosaurs!) In mathematics, we generally don't mess with premises that aren't assumed to be true, so the "soundness" of arguments isn't a consideration.

Example 6. Following is a problem taken off a homework help site on the internet (kma7 Newbie. "Math Help Forum". Jelsoft Enterprises Ltd. Date of access 9/22/09)

"Determine the validity of the next argument by using Euler circles…"

"No A is B.
Some C is A.
Therefore Some C is not B. "

To solve this problem we need to make Euler circles for A, B and C

 May or may not overlap here	Note that some of C overlaps with A (it HAS to since "Some C is A") and A is completely separate from B (since "No A is B"). So SOME C (at least the part that's in A) has to be OUTSIDE of B which means some C is NOT B. So this argument is VALID. Even if some C does overlap B (which I've shown but we don't know that it does) it still wouldn't invalidate the argument.

Homework:

Determine whether the following examples use *deductive reasoning* or *inductive reasoning*. Give a reason for your choice.

1. Numerous studies have shown that pink has a soothing effect on people with mental illness. Based on this discovery, Atascadero State Hospital painted the patient wards pink and found there was a 20% reduction in violent episodes over the course of the year.

2. All the sheep you've seen are white. You conclude that all sheep are white.

3. The Equality Property of Division states that multiplying or dividing an equation by a non-zero number won't change the solution to an equation. You solve $2x = 6$ by dividing both sides by 2, then state the solution to the original equation is $x = 3$.

4. Newton's Law of Gravity can be used to derive the path of comets. Using this law, astronomers coreectly predicted the path that Hailey's comet would take on its most recent pass around our sun.

Sketch Euler's circles to create a diagram for the following statements:

5. Some mice are blind.

6. No dogs are vicious.

7. All Math 230 students are brilliant.

8. No cars are clean. Some clean objects are expensive.

9. All cars are clean. All clean objects are expensive.

Use Euler diagrams to determine the validity of the following arguments. Show both the circles and write your conclusion.

10. No potato chips are fat-free.
All fat-free foods are low-calorie.
$\therefore$ No potato chips are low-calorie.

11. All athletes are strong.
All strong people are tall.
$\therefore$ Some tall people are athletes.

12. All philosophers are wise.
Socrates was a philosopher.
$\therefore$ Socrates was wise.

13. Some birds can fly.
All birds are dinosaurs.
$\therefore$ All dinosaurs can fly.

14. Musicians don't like garbage cans.
Fred likes garbage cans.
$\therefore$ Fred is not a musician.