Math 236: Confidence Intervals (Chapter 8)

Point Estimate: You have a "point estimate" when a single *statistic* (STAT) from a <u>sample</u> is used to estimate the *parameter* of a <u>population</u>.

A sample mean, $\bar{x}$, can be used as a point estimate of a population mean, μ

A sample proportion, $\hat{p}$, can be used as a point estimate of a population proportion, p

Margin of Error: $\pm$ M
Along with the point estimate, the margin of error creates a range of values in which the parameter might fall. It also gives you an intuitive sense of how good (precise) the estimation is.

Confidence Interval: The actual range of values in which we hope to find the population parameter.

$$\text{STAT} - \text{M} < \text{ population parameter } < \text{STAT} + \text{M}$$

Level of Confidence: This is the percentage of all the confidence intervals that could be constructed (in theory) that actually do contain (cover) the parameter. We say, for example, "We're 95% confident that the parameter is in this interval." What this does NOT mean is that there's a 95% probability the parameter is in that interval.

Confidence Intervals for a Population Mean, Population Standard Deviation Known (Section 8.1)

Assumptions:
(1) You know the population standard deviation.
(2) Sample size:
  - You either have a large sample (n $\geq$ 30)
  - Or you have a small sample (n $<$ 30) AND you know the underlying population is normal.

How can we tell from the sample that the underlying population is normal?

The sample shouldn't have outliers. If it does, we'd suspect the underlying population isn't normal.

To look for outliers, either make a boxplot of the sample or a dotplot of the sample

Formula: For a confidence level of $1-\alpha$, the confidence interval is given by

$$\left(\bar{x} - z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}, \quad \bar{x} + z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}} \right)$$

The margin of error is $M = \pm z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}$ (Remember that $\frac{\sigma}{\sqrt{n}}$ is called the "Standard Error".)

Find the critical value, $z_{\frac{\alpha}{2}}$, for different levels of confidence. (Be able to sketch the curve that illustrates the critical value and the areas that go along with it.

Confidence Level $1-\alpha$	α	$\dfrac{\alpha}{2}$	Curve	$z_{\frac{\alpha}{2}}$

Summary of $z_{\frac{\alpha}{2}}$ values:

Confidence Level	$z_{\frac{\alpha}{2}}$
90%	
95%	
99%	

How to Construct a Confidence Interval:

Example: (Section 8.1 #44) In a simple random sample of 150 households, the sample mean number of personal computers was 1.32. Assume the population standard deviation is 0.41.

By hand, using the formula, construct a 95% confidence interval for the mean number of personal computers.

Steps:

(1) Choose or identify the level of confidence.

(2) Find $z_{\frac{\alpha}{2}}$ according to the level of confidence.

(3) Write the formula for the confidence interval:

(4) Identify/find $\overline{x}$, $z_{\frac{\alpha}{2}}$, σ , and n
and fill them into the formula.

(5) Interpret! Include the point estimate,
margin of error, and confidence in your
interpretation.

Follow-up questions:

      If the sample size were 100 rather than 150, would the margin of error be larger or smaller? Explain.

      If the confidence level were 99% rather than 95%, would the margin of error be larger or smaller? Explain.

Using technology: Confidence Intervals using Minitab:

Steps: 1. Click on **Stat**, then **Basic Statistics**, then **1-Sample Z**
       2. Choose one of the following:
- IF the summary statistics are given, click **Summarized Data** and enter the values.
- IF you're using raw data, click **Samples in Columns**. You'll still need to enter the given population standard deviation.

       3. Click **Options** and enter the confidence level in the **Confidence Level** field. Click **OK**.
       4. Click **OK**.

Example: (Section 8.1 #44) In a simple random sample of 150 households, the sample mean number of personal computers was 1.32. Assume the population standard deviation is 0.41.

Use Minitab to construct a 95% confidence interval for the mean number of personal computers.

(The Minitab output you're working toward is shown below.)

One-Sample Z

```
The assumed standard deviation = 0.41

  N    Mean   SE Mean        95% CI
150   1.3200   0.0335   (1.2544, 1.3856)
```

Example: Using the raw data from the class, find a 95% confidence interval for the mean number of personal computers in Cuesta College student households.

How to Find the Required Minimum Sample Size (based on a desired margin of error)

Solve the margin of error formula for n: $\quad M = z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}} \qquad \Rightarrow \qquad n = \left(\frac{z_{\frac{\alpha}{2}} \sigma}{M} \right)^2$

Example: Considering again the simple random sample of 150 households, where the sample mean number of personal computers was 1.32 and the population standard deviation is 0.41.

How many households would have to be surveyed in order to reduce the margin error to .05?