Math 247: Theoretical Probability: The Addition Rule and Complements (Section 5.2)

Mutually Exclusive Events: *A and B are mutually exclusive if they cannot both happen.*

Example: Use your knowledge of the world and a Venn diagram to determine whether the following events are mutually exclusive or not.

(a) A student being a business major;
a student being in statistics.

M.E. **not M.E**

(b) The weather being completely sunny (no clouds);
the weather being rainy

M.E. **not M.E**

(c) A person being a rock-climber;
a person being an engineer

M.E. **not M.E**

(d) A person being 5 years old;
a person being a U.S. senator

M.E. **not M.E**

Probability Addition Rule for Mutually Exclusive Events:

$$P(A \text{ or } B) = P(A) + P(B)$$

$$P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C)$$

$$P(A \text{ or } B \text{ or } C \text{ or } D) = P(A) + P(B) + P(C) + P(D)$$

Example: A standard deck of playing cards has 52 cards, with 4 suits (hearts, spades, diamonds, clubs), 13 “kinds” (2, 3, …, 10, jacks, queens, kings, aces), and 2 colors (black clubs and spades, red diamonds and hearts).

If you draw 1 card randomly from the deck what is the probability of each of the following:

(a) The card will be a heart

Example (continued):

(b) The card will be a face card

(c) The card will be a king

(d) The card will be queen

(e) The card will be a king or a queen

(f) The card will be a king AND a queen.

Now find the probability the card will be a queen or a heart

Did the Addition Rule work in this case? Why or why not?

General Addition Rule: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Why the subtraction? P(A) and P(B) DOUBLE COUNT the outcomes which are both A and B, so the subtraction removes the double-counted outcomes.

Example: Suppose you draw 1 card from a deck of cards. Use the General Addition Rule to find the probability the card will be

(a) a jack or a red card

Example (continued):

(b) an ace or a spade

(c) a five or a nine

$P(A \text{ and } B) = 0$ if and only if A and B are mutually exclusive.

Complements of Events

Example: If the probability of rain today is 20%, what is the probability of no rain?

Complement (Negation) of an Event: A^C = the **complement** of A = "not A"

Events and their complements are automatically mutually exclusive.

Probability of the Complement of an Event: $P(A) + P(A^C) = 1$

So ….. ______________________________

Example: If the probability of getting a "lemon" (a bad new car) is .001, what is the probability you will not get a "lemon" if you buy a new car?

Use the proper notation!

Multiple Trials:

Example: Suppose you flip a coin three times. List all the possible outcomes. Then set up a table showing the number of heads and the associated probabilities.

X =				
P(X) =				

Find the following probabilities. **Use the proper notation**.

Probability of getting 2 heads.

Probability of getting no heads.

Probability of getting at least one head.

Probability of "At Least One": If X is a discrete Random Variable (more on this later!), then

$$P(X \geq 1) = 1 - P(X = 0)$$