

Math 247: Confidence Intervals for a Population Mean (Section 9.3)

The Central Limit Theorem for Means (Section 9.2) states that if a sample is selected such that the three conditions are met then the distribution of sample means follows an approximately Normal distribution, with standard error based on the population standard deviation.

Conditions that must be met for **Central Limit Theorem to apply** (page 416):

1. **Random and Independent.** The sample must be random and the observations in the sample must be independent from one another.
2. **Large Sample.** Either the underlying (target) population distribution is Normal
OR the sample size is large ($n \geq 25$)
3. **Large Population.** The population must be at least 10 times the size of the sample.

Major Problem: We almost never will know the population standard deviation! We can use the sample standard deviation as an estimate **but that introduces more variability**.

To the rescue! A new distribution curve (probability density function), **The Student's t-distribution**.

Features:

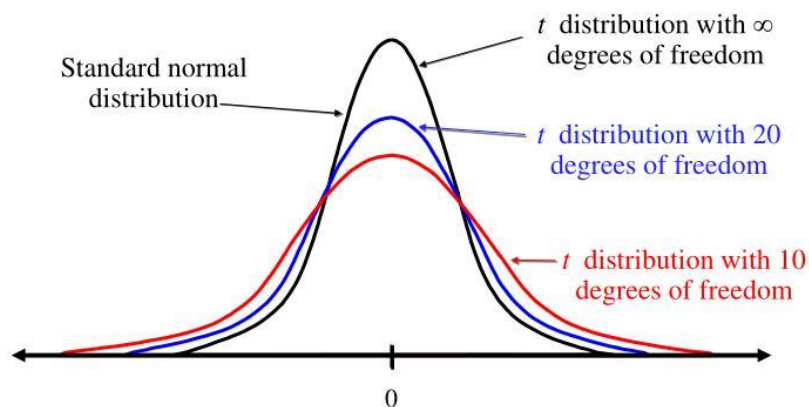
- The curve is *bell-shaped* like the normal distribution
- The curve is *wider than the normal curve* (resulting in fatter tails, hence larger P-values)
- The t-distribution has *different curves* based on sample size
- The sample size is taken into account by the “degrees of freedom” = df or

$$\text{Formula: } df = n - 1$$

Smaller sample \Rightarrow Fewer degrees of freedom \Rightarrow wider curve \Rightarrow larger P-value, larger Margin of Error!

t Distribution

The t-distribution is used when n is **small** and σ is **unknown**.



So it's easier to find statistical significance (small P-value) when you have a larger sample, which makes sense!

Example: Use the Student's t-Distribution [table](#) or StatCrunch to find the following t^* values.

Sample size is 30	Sample size is 101
Confidence level: 90% $t^* =$ _____	Confidence level: 90% $t^* =$ _____
Confidence level: 95% $t^* =$ _____	Confidence level: 95% $t^* =$ _____
Confidence level: 99% $t^* =$ _____	Confidence level: 99% $t^* =$ _____

Constructing a Confidence Interval for a Population Mean:

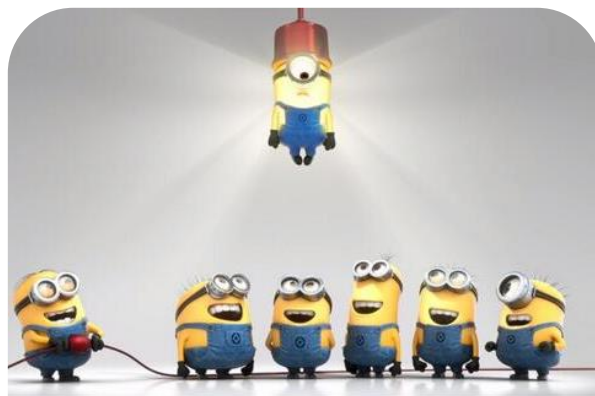
Formula: $\bar{x} \pm t^* SE_{t-dist}$

$$SE_{t-dist} = \frac{s}{\sqrt{n}}$$

Example: Lightbulb! A company has developed a new type of energy-saving lightbulb, and wants to estimate the mean lifetime of the bulbs. A simple random sample of 30 bulbs had a mean lifetime of 1020 hours with a standard deviation of 45 hours.

Construct a 95% confidence interval for the mean lifetime of all bulbs manufactured by this new process. Do this by hand, using the formula. Then use StatCrunch (steps below) to find the confidence interval and interpret the results.

Find the CI by hand



Find the confidence interval using StatCrunch:

StatCrunch steps for finding a CI of a single mean.

- Steps:
1. If you have raw data, enter it into a column in a StatCrunch worksheet.
 2. Click on **Stat**, then **T Stats**, then **One Sample**
 3. Choose one of the following:
 - **With Data**: If you have entered the data into a column then click **With Data** and select the column
 - **With Summary**: If you already have the sample mean and standard deviation, then click **With Summary** and enter the values.
 4. Enter the confidence level in the **Confidence Level** field.
 5. Click **Compute**.

One sample T summary confidence interval:

μ : Mean of population

95% confidence interval results:

Mean	Sample Mean	Std. Err.	DF	L. Limit	U. Limit
μ	1020	8.2158384	29	1003.1967	1036.8033

CI:

Interpretation:

Follow-up question:

(1) Does this result tell us how long any individual lightbulb will last?

(2) The company is going to advertise these bulbs as having an average lifespan of 1000 hours. However, they will get in legal trouble for false advertising if it was found that the average lifetime of these bulbs was actually significantly less than 1000 hours. Based on the confidence interval, should the company go ahead with its marketing plan?

Example: Nutrition. Kale is a type of cabbage that is known for its high mineral content. Suppose a lab made 6 measurements of kale to determine the calcium content (in mg). Each measurement was of 200 grams of chopped, boiled kale with the following results:

175mg 184mg 204mg 191mg 218mg

Is this raw data or summarized data? _____

If the lab wants to use this data to construct a confidence interval for how much calcium is contained in a serving kale, on average, what conditions have to be met for using the Central Limit Theorem for Means, using the t-Distribution?

1.

2.

Use StatCrunch to construct a 90% confidence interval for the mean mineral content from the data (outlier removed):

StatCrunch Confidence Interval:

Sketch the CI:

What is the margin of error?

Interpretation:

StatCrunch result for reference:

90% confidence interval results:

Variable	Sample Mean	Std. Err.	DF	L. Limit	U. Limit
Calcium	194.4	7.5670338	4	178.26824	210.53176

A nutrition website claims that that, on average, a 200 gram serving of kale has a calcium content of 175 mg. Does your work above support this claim, at the .10 level of significance? Explain.

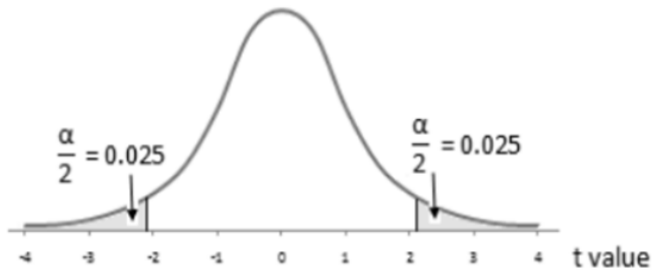
Change the confidence level to 95% in StatCrunch and graph the confidence interval by hand

95% confidence interval results:

Variable	Sample Mean	Std. Err.	DF	L. Limit	U. Limit
Calcium	194.4	7.5670338	4	173.39055	215.40945

Does this CI support the claim that, on average, kale has 175 mg of calcium per serving, at the .05 level of significance? Explain.

Student's t-Distribution Table, for finding t^* values



Confidence	90	95	95.45	99	99.73
alpha	0.1000	0.0500	0.0455	0.0100	0.0027
<i>df</i>					
1	6.314	12.706	13.968	63.657	235.784
2	2.920	4.303	4.527	9.925	19.206
3	2.353	3.182	3.307	5.841	9.219
4	2.132	2.776	2.869	4.604	6.620
5	2.015	2.571	2.649	4.032	5.507
6	1.943	2.447	2.517	3.707	4.904
7	1.895	2.365	2.429	3.499	4.530
8	1.860	2.306	2.366	3.355	4.277
9	1.833	2.262	2.320	3.250	4.094
10	1.812	2.228	2.284	3.169	3.957
11	1.796	2.201	2.255	3.106	3.850
12	1.782	2.179	2.231	3.055	3.764
13	1.771	2.160	2.212	3.012	3.694
14	1.761	2.145	2.195	2.977	3.636
15	1.753	2.131	2.181	2.947	3.586
16	1.746	2.120	2.169	2.921	3.544
17	1.740	2.110	2.158	2.898	3.507
18	1.734	2.101	2.149	2.878	3.475
19	1.729	2.093	2.140	2.861	3.447
20	1.725	2.086	2.133	2.845	3.422
21	1.721	2.080	2.126	2.831	3.400
22	1.717	2.074	2.120	2.819	3.380
23	1.714	2.069	2.115	2.807	3.361
24	1.711	2.064	2.110	2.797	3.345
25	1.708	2.060	2.105	2.787	3.330
26	1.706	2.056	2.101	2.779	3.316
27	1.703	2.052	2.097	2.771	3.303
28	1.701	2.048	2.093	2.763	3.291
29	1.699	2.045	2.090	2.756	3.280
30	1.697	2.042	2.087	2.750	3.270
40	1.684	2.021	2.064	2.704	3.199
50	1.676	2.009	2.051	2.678	3.157
60	1.671	2.000	2.043	2.660	3.130
70	1.667	1.994	2.036	2.648	3.111
80	1.664	1.990	2.032	2.639	3.096
90	1.662	1.987	2.028	2.632	3.085
100	1.660	1.984	2.025	2.626	3.077
1000	1.646	1.962	2.003	2.581	3.007
∞	1.645	1.960	2.000	2.576	3.000