

Math 247: Extra Practice Problems for Final Exam (Fall 2019)

These problems require a lot of critical thinking and connecting ideas. The Final Exam will NOT be in this format but these problems should help you connect ideas and sharpen up your analysis of some of the key ideas we've covered.

Directions: Do work (if needed) for each problem then decide which is the best of the choices given. Answers are at the end of the problem set.

- Which of the following is a key distinction between controlled experiments and observational studies?
 - More subjects are available for experiments than for observational studies.
 - Ethical constraints prevent large-scale observational studies.
 - Experiments are less costly to conduct than observational studies.
 - An experiment can show a direct cause-and-effect relationship, whereas an observational study cannot.
 - Tests of significance cannot be used on data collected from an observational study.

- A manufacturer of balloons claims that p , the proportion of its balloons that burst when inflated to a diameter of up to 12 inches, is no more than 0.05. Some customers have complained that the balloons are bursting more frequently. If the customers want to conduct an experiment to test the manufacturer's claim, which of the following hypotheses would be appropriate?
 - $H_0: p \neq 0.05$, $H_a: p = 0.05$
 - $H_0: p = 0.05$, $H_a: p > 0.05$
 - $H_0: p = 0.05$, $H_a: p \neq 0.05$
 - $H_0: p = 0.05$, $H_a: p < 0.05$
 - $H_0: p < 0.05$, $H_a: p = 0.05$

- Lauren is enrolled in a very large college calculus class. On the first exam, the class mean was 75 and the standard deviation was 10. On the second exam, the class mean was 70 and the standard deviation was 15. Lauren scored 85 on both exams. Assuming the scores on each exam were approximately normally distributed, on which exam did Lauren score better relative to the rest of the class?
 - She scored much better on the first exam.
 - She scored much better on the second exam.
 - She scored about equally well on both exams.
 - It is impossible to tell because the class size is not given.
 - It is impossible to tell because the correlation between the two sets of exam scores is not given.

- Omit

- Omit

6. The correlation between two scores X and Y equals 0.8. We know from this that

- (A) The slope of the regression line would be close to zero.
- (B) There is a negative trend between X and Y
- (C) In a scatterplot of X and Y , the dots would be a “cloud” and show little to no trend
- (D) 80% of the variation in Y can be explained by X .
- (E) 64% of the variation in Y can be explained by X

7. Suppose that the distribution of a set of scores has a mean of 47 and a standard deviation of 14. If 4 is added to each score, what will be the mean and the standard deviation of the distribution of new scores?

	<u>Mean</u>	<u>Standard Deviation</u>
(A)	51	14
(B)	51	18
(C)	47	14
(D)	47	16
(E)	47	18

8. A test engineer wants to estimate the mean gas mileage μ (in miles per gallon) for a particular model of automobile. Eleven of these cars are subjected to a road test, and the gas mileage is computed for each car.

A dotplot of the 11 gas-mileage values is roughly symmetrical and has no outliers. The mean and standard deviation of these values are 25.5 and 3.01, respectively. Assuming that these 11 automobiles can be considered a simple random sample of cars of this model, which of the following is a correct statement?

- (A) A 95% confidence interval for μ is $25.5 \pm 2.228 \times \frac{3.01}{\sqrt{11}}$.
- (B) A 95% confidence interval for μ is $25.5 \pm 2.201 \times \frac{3.01}{\sqrt{11}}$.
- (C) A 95% confidence interval for μ is $25.5 \pm 2.228 \times \frac{3.01}{\sqrt{10}}$.
- (D) A 95% confidence interval for μ is $25.5 \pm 2.201 \times \frac{3.01}{\sqrt{10}}$.
- (E) The results cannot be trusted; the sample is too small.

9. A volunteer for a mayoral candidate's campaign periodically conducts polls to estimate the proportion of people in the city who are planning to vote for this candidate in the upcoming election. Two weeks before the election, the volunteer plans to double the sample size in the polls. The main purpose of this is to

- (A) reduce nonresponse bias
- (B) reduce the effects of confounding variables
- (C) reduce bias due to the interviewer effect
- (D) decrease the variability in the population
- (E) decrease the standard deviation of the sampling distribution of the sample proportion

10. Omit

11. The following two-way table resulted from classifying each individual in a random sample of residents of a small city according to level of education (with categories "earned at least a high school diploma" and "did not earn a high school diploma") and employment status (with categories "employed full time" and "not employed full time").

	Employed full time	Not employed full time	Total
Earned at least a high school diploma	52	40	92
Did not earn a high school diploma	30	35	65
Total	82	75	157

If the null hypothesis of no association between level of education and employment status is true, which of the following expressions gives the expected number who earned at least a high school diploma and who are employed full time?

- (A) $\frac{92 \cdot 52}{157}$
- (B) $\frac{92 \cdot 82}{157}$
- (C) $\frac{82 \cdot 52}{92}$
- (D) $\frac{65 \cdot 52}{92}$
- (E) $\frac{92 \cdot 52}{82}$

12. The manager of a factory wants to compare the mean number of units assembled per employee in a week for two new assembly techniques. Two hundred employees from the factory are randomly selected and each is randomly assigned to one of the two techniques. After teaching 100 employees one technique and 100 employees the other technique, the manager records the number of units each of the employees assembles in one week. Which of the following would be the most appropriate inferential statistical test in this situation?

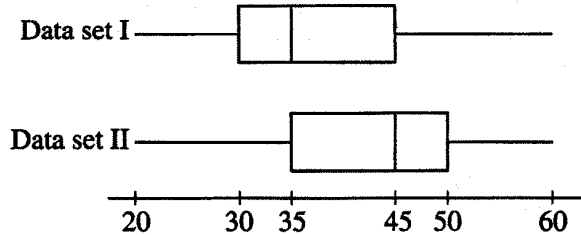
- (A) ANOVA
- (B) Two-sample t-test
- (C) Paired t-test
- (D) Chi-square goodness-of-fit test
- (E) One-sample t-test

13. A random sample has been taken from a population. A statistician, using this sample, needs to decide whether to construct a 90 percent confidence interval for the population mean or a 95 percent confidence interval for the population mean. How will these intervals differ?

- (A) The 90 percent confidence interval will not be as wide as the 95 percent confidence interval.
- (B) The 90 percent confidence interval will be wider than the 95 percent confidence interval.
- (C) Which interval is wider will depend on how large the sample is.
- (D) Which interval is wider will depend on whether the sample is unbiased.
- (E) Which interval is wider will depend on whether a z -statistic or a t -statistic is used.

14. The boxplots shown below summarize two data sets, I and II. Based on the boxplots, which of the following statements about these two data sets CANNOT be justified?

- (A) The range of data set I is equal to the range of data set II.
- (B) The interquartile range of data set I is equal to the interquartile range of data set II.
- (C) The median of data set I is less than the median of data set II.
- (D) Data set I and data set II have the same number of data points.
- (E) About 75% of the values in data set II are greater than or equal to about 50% of the values in data set I.



15. A high school statistics class wants to conduct a survey to determine what percentage of students in the school would be willing to pay a fee for participating in after-school activities. Twenty students are randomly selected from each of the freshman, sophomore, junior, and senior classes to complete the survey. This plan is an example of which type of sampling?

- (A) Cluster
- (B) Convenience
- (C) Simple random
- (D) Stratified random
- (E) Systematic

16. Jason wants to determine how age and gender are related to political party preference in his town. Voter registration lists are stratified by gender and age-group. Jason selects a simple random sample of 50 men from the 20 to 29 age-group and records their age, gender, and party registration (Democratic, Republican, neither). He also selects an independent simple random sample of 60 women from the 40 to 49 age-group and records the same information. Of the following, which is the most important observation about Jason's plan?

- (A) The plan is well conceived and should serve the intended purpose.
- (B) His samples are too small.
- (C) He should have used equal sample sizes.
- (D) He should have randomly selected the two age groups instead of choosing them nonrandomly.
- (E) He will be unable to tell whether a difference in party affiliation is related to differences in age or to the difference in gender.

17. A least squares regression line was fitted to the weights (in pounds) *versus* age (in months) of a group of many young children. The equation of the line is

$$\hat{y} = 16.6 + 0.65t,$$

where \hat{y} is the predicted weight and t is the age of child. A 20-month-old child in this group has an actual weight of 25 pounds. Which of the following is the residual weight, in pounds, for this child?

- (A) -7.85
- (B) -4.60
- (C) 4.60
- (D) 5.00
- (E) 7.85

18. Which of the following statements is (are) true about the t -distribution with k degrees of freedom?

- I. The t -distribution is symmetric.
- II. The t -distribution with k degrees of freedom has less spread than the t -distribution with $k + 1$ degrees of freedom.
- III. The t -distribution has a larger spread than the standard normal (z) distribution.

- (A) I only
- (B) II only
- (C) III only
- (D) I and II
- (E) I and III

19. A geneticist hypothesizes that half of a given population will have brown eyes and the remaining half will be split evenly between blue- and green-eyed people. In a random sample of 60 people from this population, the individuals are distributed as shown in the table below. What is the value of the χ^2 statistic for the goodness of fit test on these data?

- (A) Less than 1
- (B) At least 1, but less than 10
- (C) At least 10, but less than 20
- (D) At least 20, but less than 50
- (E) At least 50

Brown Eyes	Green Eyes	Blue Eyes
34	15	11

20. A small town employs 34 salaried, nonunion employees. Each employee receives an annual salary increase of between \$500 and \$2000 based on a performance review by the mayor's staff. Some employees are members of the mayor's political party, and the rest are not.

Students at the local high school form two lists, A and B, one for the raises granted to employees who are in the mayor's party, and the other for raises granted to employees who are not. They want to display a graph (or graphs) of the salary increases in the student newspaper that readers can use to judge whether the two groups of employees have been treated in a reasonably equitable manner.

Which of the following displays is least likely to be useful to readers for this purpose?

- (A) Back-to-back stemplots of A and B
- (B) Scatterplot of B *versus* A
- (C) Parallel boxplots of A and B
- (D) Histograms of A and B that are drawn to the same scale
- (E) Dotplots of A and B that are drawn to the same scale

21. Omit

22. A study of existing records of 27,000 automobile accidents involving children in Michigan found that about 10 percent of children who were wearing a seatbelt (group SB) were injured and that about 15 percent of children who were not wearing a seatbelt (group NSB) were injured. Which of the following statements should NOT be included in a summary report about this study?

- (A) Driver behavior may be a potential confounding factor.
- (B) The child's location in the car may be a potential confounding factor.
- (C) This study was not an experiment, and cause-and-effect inferences are not warranted.
- (D) This study demonstrates clearly that seat belts save children from injury.
- (E) Concluding that seatbelts save children from injury is risky, at least until the study is independently replicated.

23. Which of the following statements is true for two events, each with probability greater than 0?

- (A) If the events are mutually exclusive, they must be independent.
- (B) If the events are independent, they must be mutually exclusive.
- (C) If the events are not mutually exclusive, they must be independent.
- (D) If the events are not independent, they must be mutually exclusive.
- (E) If the events are mutually exclusive, they cannot be independent.

24. A consulting statistician reported the results from a learning experiment to a psychologist. The report stated that on one particular phase of the experiment a statistical test result yielded a p-value of 0.24. Based on this p-value, which of the following conclusions should the psychologist make?

- (A) The test was statistically significant because a p-value of 0.24 is greater than a significance level of 0.05.
- (B) The test was statistically significant because $p = 1 - 0.24 = 0.76$ and this is greater than a significance level of 0.05.
- (C) The test was not statistically significant because $2 \text{ times } 0.24 = 0.48$ and that is less than 0.5.
- (D) The test was not statistically significant because, if the null hypothesis is true, one could expect to get a test statistic at least as extreme as that observed 24% of the time.
- (E) The test was not statistically significant because, if the null hypothesis is true, one could expect to get a test statistic at least as extreme as that observed 76% of the time.

25. A new medication has been developed to treat sleep-onset insomnia (difficulty in falling asleep). Researchers want to compare this drug to a drug that has been used in the past by comparing the length of time it takes subjects to fall asleep. Of the following, which is the best method for obtaining this information?

- (A) Have subjects choose which drug they are willing to use, then compare the results.
- (B) Assign the two drugs to the subjects on the basis of their past sleep history without randomization, then compare the results.
- (C) Give the new drug to all subjects on the first night. Give the old drug to all subjects on the second night. Compare the results.
- (D) Randomly assign the subjects to two groups, giving the new drug to one group and no drug to the other group, and then compare the results.
- (E) Randomly assign the subjects to two groups, giving the new drug to one group and the old drug to the other group, and then compare the results.

26. Omit

27. Omit

28. Omit

29. The analysis of a random sample of 500 households in a suburb of a large city indicates that a 98 percent confidence interval for the mean family income is (\$41,300, \$58,630). Could this information be used to conduct a test of the null hypothesis $H_0: \mu = 40,000$ against the alternative hypothesis $H_a: \mu \neq 40,000$ at the $\alpha = 0.02$ level of significance?

- (A) No, because the value of σ is not known.
- (B) No, because it is not known whether the data are normally distributed.
- (C) No, because the entire data set is needed to do this test.
- (D) Yes, since \$40,000 is not contained in the 98 percent interval, the null hypothesis would be rejected in favor of the alternative, and it could be concluded that the mean family income is significantly different from \$40,000 at the $\alpha = 0.02$ level.
- (E) Yes, since \$40,000 is not contained in the 98 percent interval, the null hypothesis would not be rejected, and it could be concluded that the mean family income is significantly different from \$40,000 at the $\alpha = 0.02$ level.

30. The population $\{2, 3, 5, 7\}$ has mean $\mu = 4.25$ and standard deviation $\sigma = 1.92$. When sampling with replacement, there are 16 different possible ordered samples of size 2 that can be selected from this population. The mean of each of these 16 samples is computed. For example, 1 of the 16 samples is (2, 5), which has a mean of 3.5. The distribution of the 16 sample means has its own mean $\mu_{\bar{x}}$ and its own standard deviation $\sigma_{\bar{x}}$. Which of the following statements is true?

- (A) $\mu_{\bar{x}} = 4.25$ and $\sigma_{\bar{x}} = 1.92$
- (B) $\mu_{\bar{x}} = 4.25$ and $\sigma_{\bar{x}} > 1.92$
- (C) $\mu_{\bar{x}} = 4.25$ and $\sigma_{\bar{x}} < 1.92$
- (D) $\mu_{\bar{x}} > 4.25$
- (E) $\mu_{\bar{x}} < 4.25$

31. A wildlife biologist is interested in the relationship between the number of chirps per minute for crickets (y) and temperature. Based on the collected data, the least squares regression line is $\hat{y} = 10.53 + 3.41x$, where x is the number of degrees Fahrenheit by which the temperature exceeds 50° . Which of the following best describes the meaning of the slope of the least squares regression line?

- (A) For each increase in temperature of 1° F, the estimated number of chirps per minute increases by 10.53.
- (B) For each increase in temperature of 1° F, the estimated number of chirps per minute increases by 3.41.
- (C) For each increase of one chirp per minute, there is an estimated increase in temperature of 10.53° F.
- (D) For each increase of one chirp per minute, there is an estimated increase in temperature of 3.41° F.
- (E) The slope has no meaning because the units of measure for x and y are not the same.

32. Omit

33. An engineer for the Allied Steel Company has the responsibility of estimating the mean carbon content of a particular day's steel output, using a random sample of 15 rods from that day's output. The actual population distribution of carbon content is not known to be normal, but graphic displays of the engineer's sample results indicate that the assumption of normality is not unreasonable. The process is newly developed, and there are no historical data on the variability of the process. In estimating this day's mean carbon content, the primary reason the engineer should use a t -confidence interval rather than a z -confidence interval is because the engineer

- (A) is estimating the population mean using the sample mean.
- (B) is using the sample variance as an estimate of the population variance.
- (C) is using data, rather than theory, to judge that the carbon content is normal.
- (D) is using data from a specific day only.
- (E) has a small sample, and a z -confidence interval should never be used with a small sample.

34. Each of 100 laboratory rats has available both plain water and a mixture of water and caffeine in their cages. After 24 hours, two measures were recorded for each rat: the amount of caffeine the rat consumed, X , and the rat's blood pressure, Y . The correlation between X and Y was 0.428. Which of the following conclusions is justified on the basis of this study?

- (A) The correlation between X and Y in the population of rats is also 0.428.
- (B) If the rats stop drinking the water/caffeine mixture, this would cause a reduction in their blood pressure.
- (C) About 18 percent of the variation in blood pressure can be explained by a linear relationship between blood pressure and caffeine consumed.
- (D) Rats with lower blood pressure do not like the water/caffeine mixture as much as do rats with higher blood pressure.
- (E) Since the correlation is not very high, the relationship between the amount of caffeine consumed and blood pressure is not linear.

35. Omit

36. Omit

37. A simple random sample procedure produces a sample mean, \bar{x} , of 15. A 95 percent confidence interval for the corresponding population mean is 15 ± 3 . Which of the following statements must be true?

- (A) Ninety-five percent of the population measurements fall between 12 and 18.
- (B) Ninety-five percent of the sample measurements fall between 12 and 18.
- (C) If 100 samples were taken, 95 of the sample means would fall between 12 and 18.
- (D) $P(12 \leq \bar{x} \leq 18) = 0.95$
- (E) If $\mu = 19$, this \bar{x} of 15 would be unlikely to occur.

38. Suppose that public opinion in a large city is 35 percent against increasing taxes to support the public school system. If a random sample of 500 people from this city are interviewed, what is the approximate probability that more than 200 of these people will be against increasing taxes? Which of the following set-ups would answer the question?

(A) $\binom{500}{200} (0.65)^{200} (0.35)^{300}$

(B) $\binom{500}{200} (0.35)^{200} (0.65)^{300}$

(C) $P \left(z > \frac{0.40 - 0.65}{\sqrt{\frac{(0.65)(0.35)}{500}}} \right)$

(D) $P \left(z > \frac{0.40 - 0.35}{\sqrt{\frac{(0.40)(0.60)}{500}}} \right)$

(E) $P \left(z > \frac{0.40 - 0.35}{\sqrt{\frac{(0.35)(0.65)}{500}}} \right)$

39. As lab partners, Sally and Betty collected data for a significance test. Both calculated the same z -test statistic, but Sally found the results were significant at the $\alpha = 0.05$ level while Betty found that the results were not. When checking their results, the women found that the only difference in their work was that Sally used a two-sided test, while Betty used a one-sided test. Which of the following could have been their test statistic?

- (A) -1.980
- (B) -1.690
- (C) 1.340
- (D) 1.690
- (E) 1.780

40. A student working on a history project decided to find a 95 percent confidence interval for the difference in mean age at the time of election to office for former American Presidents versus former British Prime Ministers. The student found the ages at the time of election to office for the members of both groups, which included all of the American Presidents and all of the British Prime Ministers, and used StatCrunch to find the 95 percent confidence interval based on the t -distribution. This procedure is not appropriate in this context because

- (A) the sample sizes for the two groups are not equal.
- (B) the entire population was measured in both cases, so the actual difference in means can be computed and a confidence interval should not be used
- (C) election to office take place at different intervals in the two countries, so the distribution of ages cannot be the same
- (D) ages at the time of election to office are likely to be skewed rather than bell-shaped, so the assumptions for using this confidence interval formula are not valid
- (E) ages at the time of election to office are likely to have a few large outliers, so the assumptions for using this confidence interval formula are not valid

Answer Key

1. D	9. E	17. B	25. E	33. B
2. B	10. D	18. E	26. D	34. C
3. C	11. B	19. B	27. A	35. C
4. A	12. B	20. B	28. E	36. B
5. D	13. A	21. A	29. D	37. E
6. E	14. D	22. D	30. C	38. E
7. A	15. D	23. E	31. B	39. A
8. A	16. E	24. D	32. E	40. B