Math 247: Final Exam **Name:** KEY
(100 pts)

1. (25 pts) **Vitamin C** A study (double-blind) was done investigating the therapeutic value of vitamin C (ascorbic acid) for treating common colds. The study (done in 1971 by Linus Pauling) was conducted during a 2-week period on a sample of 279 school children in a skiing camp in the Swiss Alps. The participants were split into two groups (assume random assignment). In one group the kids took 1 gram of vitamin C per day; in the other group, the kids took a placebo. At the end of two weeks the researchers assessed who had gotten a cold and who hadn't.

2 A. Is the sample (using the kids at the ski-camp) considered a simple random sample from the population of all school children? Yes No

	Cold	No Cold	Total
Placebo	31	109	140
Vitamin C	17	122	139
	48		279

The name for this type of sample is

"SAMPLE OF Convenience"

2 B. Would it be correct to apply the findings of this study to all school age children? Why or why not?

No, this is not a random sample of school age children; not all children were equally likely to be chosen.

2 C. Is this study a Controlled, Randomized Experiment or an Observational Study? How can you tell?

This is a controlled, randomized experiment, due to random assignment to the groups, one with vitamin C and one with placebo (the control).

2 D. Which of the following hypothesis tests could be used to see whether there was a significant difference in the proportion of of kids who got colds, based on whether or not they took Vitamin C? (Circle the answer)

z-Test for Two Proportions	**One Sample t-Test**
Two Sample t-Test	**Paired Difference Test**

Success = get a cold

Parking Lot:

Vitamin C: $x_1 = 17$, $n_1 = 139$, $\hat{p}_1 = \frac{17}{139} = .122$

Placebo: $x_2 = 31$, $n_2 = 140$, $\hat{p}_2 = \frac{31}{140} = .221$

4 E. If Dr. Pauling wanted to test whether proportion of kids who got colds (in that order) in the Vitamin C group was significantly less than in the Placebo group what would the hypotheses be?

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Write the hypothesis using words and symbols.

p = proportion of kids who would get colds in the population

p_1 = proportion of colds in population, if taking Vitamin C

p_2 = proportion of colds in population if taking placebo.

H_0: $p_1 = p_2 \quad p_1 - p_2 = 0$

There is no diffence, in population, in proportion of cold whether they take Vit. C or a placebo.

H_a: $p_1 < p_2 \Rightarrow p_1 - p_2 < 0$

The population proportion of colds is less in the Vitamin C group.

3 F. List the conditions you must check to use the test, then explain how they are or are not satisfied or assumptions you will have to make.

1. Random sample from population of all school kids?
No - this is not a random sample!

WITHIN & BETWEEN Independent observations? Probably not. The kids will interact and impact each other's health, and not uniformly

2. Large sample? $\hat{P} = \frac{48}{279} = .172$ (pooled proportion)

Vit. C: $E_1 = n_1\hat{P} = 139(.172) = 23.9 \geq 10$? Yes

Placebo: $E_2 = n_2\hat{P} = 140(.172) = 24.1 \geq 10$? Yes

3. Large population? If population is all school-aged kids, then yes Pop $\geq 10 \cdot$ Sample
If population is just kids who go to Ski Camps, then no, Pop $\ngeq 10 \cdot$ Sample.

2 G. Fill in the Compute step result from StatCrunch:

Hypothesis test results:

Difference	Count1	Total1	Count2	Total2	Sample Diff.	Std. Err.	Z-Stat	P-value
$p_1 - p_2$	17	139	31	140	−.099	.04519	−2.193	.0141

3 H. What is the conclusion of the hypothesis test? Include whether you can conclude a cause-and-effect relationship in your answer.

p-value = .0141 < .05 = α (default)
Reject Ho, accept Ha

We have convincing evidence that there were (statiscally) significantly lower proportion of colds in the Vitamin C group as compared to the placebo group. Since this was a randomized, controlled experement, we can conclude the Vitamin C caused the reduction in colds.

2 I. Fill in the 95% confidence interval from StatCrunch:

95% confidence interval results:

Difference	Count1	Total1	Count2	Total2	Sample Diff.	Std. Err.	L. Limit	U. Limit
$p_1 - p_2$	17	139	31	140	−.099	.04476	−.187	−.011

3 J. Interpret the confidence interval in the context of the problem. Be very specific about what it tells you about kids, colds, and placebo vs. Vitamin C.

We are 95% confident that Vitamin C would reduce colds in this population anywhere between 1.1% and 18.7%. In other words, the difference between the population proportion of colds is 1.1% and 18.7%, with 95% confidence. Since ZERO is not in the CI, we have evidence that there was a significant reduction in colds.

2. (25 pts) **Quality Control.** A package of Diamond roasted almonds is supposed to contain 8 ounces. The Quality Control engineer, Janine, wants to make sure the machines are filling the packages correctly, i.e., she wants to test whether the mean weight is different from 8 ounces. She sets the significance level to .05, then she draws a random sample of 25 packages and finds that the sample has a mean of 7.8 ounces with a standard deviation of 0.7 ounces.

Parking Lot
$n = 25$ – one sample!
$\bar{x} = 7.8$ oz.
$S = .7$ oz
$\mu_0 = 8$ oz

2 A. What hypothesis test should she use? (Circle your answer)

z-Test for Two Proportions **One Sample t-Test**

Two Sample t-Test **Paired Difference Test**

3 B. What are the hypotheses? Write them using math symbols then describe them, in words.

$H_0: \mu = 8$ The average weight for ALL the packages is 8 ounces (machine is working correctly.

$H_a: \mu \neq 8$ The average weight for ALL is not 8 oz. The machine is not working correctly.

1 C. Is this a one-tailed or two-tailed test? One-tailed Two-tailed

2 D. Why is the two-tailed test considered more strict (harder to get a significant result)? (Circle your answer)
(a) Because the P-value for the two-tailed test is half the P-value for the one-tailed test.
(b) Because the P-value for the two-tailed test is two times the P-value for the one-tailed test.
(c) Because the P-value doesn't matter for a two-tailed test.
(d) Because two-tails are twice as much fun!

2 E. List the conditions she must check to use the test, then explain how they are or are not satisfied or assumptions she will have to make.

1. Random sample? Yes – stated. (Pop = ALL packets)
Independent observations? Assume – reasonable.

2. Large sample OR Normal distribution of weights?
$n = 25 \geq 25 \Rightarrow$ Large sample ✓

2 F. Fill in the Compute step result from StatCrunch:

Hypothesis test results:

Mean	Sample Mean	Std. Err.	DF	T-Stat	P-value
μ	7.8	0.14	24	–1.42857	0.166

1 G. Why is the t-statistic you found in the previous answer a negative value?

(a) Because she found a significant result.
(b) Because she did not find a significant result.
(c) Because the sample mean is less than the null.
(d) Because she made a mistake.

3 H. Interpret the results from the hypothesis test.

P-value = .166 > .05 = α
Fail to reject the null.
We do not have evidence that the true mean weight of all the packages is significantly different from 8 ounces.
(Janine would not decide to adjust the machines based on this – they seem to be working fine!)

2 I. Fill in the confidence interval calculation from StatCrunch:

95% confidence interval results:

Mean	Sample Mean	Std. Err.	DF	L. Limit	U. Limit
μ	7.8	0.14	24	7.51105	8.08895

3 J. Interpret the confidence interval in the context of the problem.

We are 95% confident that the true mean weight of all the packages is between 7.5 ounces and 8.1 ounces.

3 K. Does the confidence interval support the null or the alternative hypothesis? Explain how you can tell.

It supports the null since the CI includes 8 ounces as a possibility.

3. (25 pts) **Marketing.** Big Foods Grocery has two grocery stores located in Johnston City. One store is located on First Street and the other on Main Street and each is run by a different manager. Each manager claims that her store's layout maximizes the amounts customers will purchase on impulse. Both managers surveyed a sample of 20 of their customers and asked them how much more they spent than they planned to (so impulse buys).

The average impulse spending at First Street was \$15.92, with a standard deviation of \$4.15. The average impulse spending at Main Street was \$24.75 with a standard deviation of \$6.43.

5 A. What is the research question for this situation?

Does a store's layout link to* how much impulse buying shoppers will do, on average.

* this should be "linked to" or "is associated with" rather than "affect" since this is an Observational Study.

What is the independent variable? Store Layout

What is the dependent variable? Impulse spending (dollars)

Is this an observational study or controlled experiment? Observational Experiment

Is this design balanced or unbalanced? Balanced Unbalanced

20 in each sample

7 B. Which of the hypothesis tests would you use to determine whether the average amount of impulse spending at First Street was significantly different from the average amount of spending at Main Street?

z-Test for Two Proportions	**One Sample t-Test**		
Two Sample t-Test	**Paired Difference Test**		

Parking Lot

First St.	Main St.
$n_1 = 20$	$n_2 = 20$
$\bar{x}_1 = 15.92$	$\bar{x}_2 = 24.75$
$s_1 = 4.15$	$s_2 = 6.43$

2 C. What are the hypotheses? Write them using math symbols then describe them, in words.

$H_0: \mu_{First} - \mu_{main} = 0$ On average, shoppers spend the same on impulse buys, regardless of the store.

$H_a: \mu_{First} - \mu_{Main} \neq 0$ On average, impulse buying is different at the two stores.

3 D. List the conditions you must check to use the test, then explain how they are or are not satisfied or assumptions you will have to make.

1. Random samples? Not mentioned, probably not a random sample!
   Independent observations within each sample? Assume each shopper didnt influence others.
2. Large sample OR normal populations?
   $n_1 = 20 < 25$, $n_2 = 20 < 25$ Small samples so assume \$ spent on impulse buys is normally distributed.
3. Independence between groups? Assume shoppers go to only 1 of the stores.

2 E. Fill in the Compute step result from StatCrunch:

Hypothesis test results:

Difference	Sample Diff.	Std. Err.	DF	T-Stat	P-value
$\mu_1-\mu_2$	−8.83	1.7112481	32.5	−5.1599766	<.0001

Note: You may have positive values if you put Main Street first.

3 F. Interpret the results from the hypothesis test.

P-value < .0001 < .05 = α

Reject Ho, accept Ha

We have strong evidence that there is a significant difference in the amount of money spent, on average, on impulse buys between the two stores.

2 G. Fill in the confidence interval calculation from StatCrunch:

95% confidence interval results:

Difference	Sample Diff.	Std. Err.	DF	L. Limit	U. Limit
$\mu_1-\mu_2$	−8.83	1.7112481	32.5	−12.32	−5.35

Note: You may have positive values for the limits

2 H. Interpret the confidence interval in the context of the problem.

We are 95% confident that the true difference in average impulse spending between the 2 stores, is between \$5.35 and \$12.32.

Another way to say this is that, on average, Main Street shoppers spend between \$5.35 and \$12.32 MORE than First Street shoppers.

2 I. Does the confidence interval support the null or the alternative hypothesis? Explain how you can tell.

Yes! Since the CI does not capture 0, we can exclude the possibility that there is ZERO difference (no difference) in average impulse spending at the 2 locations.

2 J. If there was a significant difference between impulse spending, on average, between the two stores, could you conclude that the store layout CAUSED the extra impulse buying at one store? Briefly explain.

No, this is an observational study, not a randomized controlled experiment. So many other factors could explain why shoppers spent more, on average, at the Main Street store.

1 K. Describe one possible confounder for this study. —Answers will vary

Location: This would influence who goes to which store.

Main street could be in a wealthier part of town, for instance.

4. (25 pts) **Drug Testing.** A study is designed to evaluate a new drug that may lower cholesterol. Fifty patients agree to participate in the study and each is asked to take the new drug for 6 weeks. Before starting the treatment, each patient's total cholesterol level is measured (BASELINE data). After taking the drug for 6 weeks, each patient's total cholesterol level is measured again (6 WEEKS data).

2 A. What is the independent variable? Medication (new drug)

What is the dependent variable? Cholesterol level

2 B. What hypothesis test would you use for this study?

z-Test for Two Proportions **One Sample t-Test**

Two Sample t-Test **Paired Difference Test** (Before-and-after)

4 C. If you wanted to test whether the subjects' cholesterol levels, on average, were lower after 6 weeks on the drug, what should the hypotheses be? Write them in math symbols and in words.

$H_0: \mu_D = 0$ (or $\mu_1 - \mu_2 = 0$) There is ZERO difference, on average, in cholesterol levels before and after in ALL such patients.

D = difference in cholesterol levels from baseline to 6-weeks later.
μ_D = mean difference

$H_a: \mu_D > 0$ or $\mu_1 - \mu_2 > 0$ The cholesterol levels would be lower, on average, after 6 weeks, for all such patients.

3 D. List the conditions you must check to use the test, then explain how they are or are not satisfied or assumptions you will have to make.

1. Random sample? No info on sample, so assume.
Independent observations? Assume no connection
Within groups between patients

2. Large sample? Yes, $n = 50 \geq 25$
3. Dependent observations between groups?
Yes, before-and-after data means
patient data is connected (dependent)

2 E. The confidence interval is for the difference between Baseline and 6 Weeks is given below: possible mean difference

99% confidence interval results:

Difference	Mean	Std. Err.	DF	L. Limit	U. Limit
Baseline - 6 Weeks	16.933333	3.657	49	6.05	27.82

4 Interpret this confidence interval in the context of the problem. Be very specific about mean cholesterol level at Baseline vs 6 weeks later.

We are 99% confident that the mean difference in cholesterol levels for patients (ALL such patients) from Baseline to after 6 weeks on the drug is between 6.05 and 27.82 (units are mg/dL - not given in problem)
Mean reduction is between 6.05 and 27.82 mg/dL over the 6-week period.
Note: A POSITIVE difference indicates Baseline values must have been higher (larger - smaller = positive)

3 F. Based on the confidence interval above, what would your conclusion be? (Circle the answer.)

(a) There is no significant difference between the mean Baseline and After 6 Weeks cholesterol levels.
(b) There was a significant increase in cholesterol levels after 6 weeks on the drug.
(c) There was a significant decrease in cholesterol levels after 6 weeks on the drug.
(d) No conclusion can be made from this information.

4 G. What is a major flaw in the design of this study? Circle all that apply.

(a) There is no placebo group for comparison
(b) The sample size is considered small.
(c) There are potential confounders, like diet and exercise, that could have impacted the cholesterol levels.
(d) None of the above.

3 H. Based on the confidence interval and the design of the study, we can conclude which of the following.

(a) The drug causes a reduction in cholesterol levels, on average.
(b) The drug does not cause a reduction in cholesterol levels, on average.
(c) There was an association between the drug and lower cholesterol but we can't say the drug caused it.
(d) No conclusion can be made.

Extra Credit: (25 points)

Sound and Learning. Maddie is doing a study where she predicts that students will learn most effectively with a constant background sound, as opposed to an unpredictable sound or no sound at all.

She randomly selects 30 students for the study then randomly assigns them into three groups of 10.

All students study a passage of text for 30 minutes. Those in group 1 study with background sound at a constant volume in the background. Those in group 2 study with noise that changes volume periodically. Those in group 3 study with no sound at all. After studying, all students take a 10 point multiple choice test over the material. Their scores follow:

group	**test scores**									
1) constant sound	7	4	6	8	6	6	2	9	8	10
2) random sound	5	5	3	4	4	7	2	2	5	4
3) no sound	2	4	7	1	2	1	5	5	6	5

Here are the Summary Statistics for this data:

Column	n	Mean	Std. Dev.	Std. Error
Constant	10	6.6	2.3664319	0.74833148
Random	10	4.1	1.5238839	0.48189441
No Sound	10	3.8	2.1499354	0.67986927

1 A. What is the research question for this situation?
Does type of background sound affect student learning?

3 B. What is the independent variable? Type of Sound

What is the dependent variable? Score on test

Is this an observational study or controlled experiment? Controlled experiment

2 C. Based on the sample means, it appears that which way of studying might be the best? Constant Sound

Since this is just sample data, we can't say these results would apply to everyone because the differences we see between the groups may be due to just Sampling variability.

3 D. Which of the hypothesis tests would you use for this situation? ANOVA

What are the hypotheses? Write them in symbols, then state what they mean in the context of the problem.

$H_0: \mu_{constant} = \mu_{random} = \mu_{control}$

Average test score would be the same regardless of type of sound, for ALL people in the population

H_a: At least one mean is different

At least one mean test score will be higher or lower than the others.

4 E. List the conditions she must check to use the test, then explain how they are or are not satisfied or assumptions she will have to make.

1. Random samples? Yes - stated
   Independent observations WITHIN groups? Assume
2. Independence BETWEEN groups? Assume
3. Large samples? No! $n_1 = n_2 = n_3 = 10 \le 25$
   so, normal populations? Assume test scores would be normally distributed
4. Equal Variances? Assume

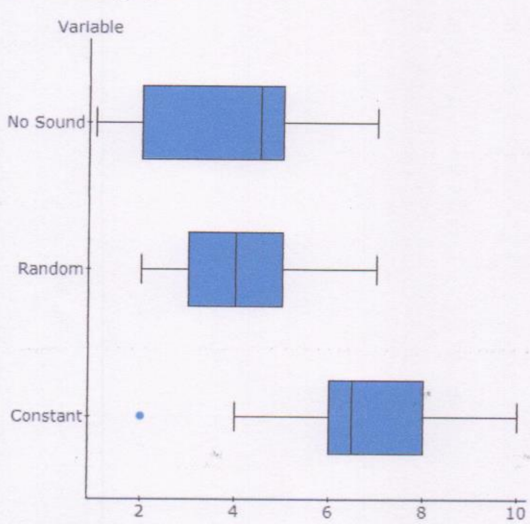
2 F. To check the condition of about population variances, Maddie examined the standard deviations (SD's) of the three groups (given at beginning of problem).

Which conclusion should she make? (Circle the answer.)

(a) The SD's are unequal so the population variances must also be unequal.
(b) The SD's are equal so the population variances must also be equal.
(c) One of the SD's is more than two times another, so she shouldn't assume equal population variances.
(d) None of the SD's is more than two times another, so she can assume equal population variances.
(e) She can't tell anything about the population from the sample data.

2 G. To check the condition of normality, Maddie graphed boxplots of the sample data.

Here is the graph:



What can Maddie tell from the boxplots? (Circle the answer.)

(a) The boxplots show the populations are perfectly symmetric, with no skewing, so she can proceed.

(b) The boxplots show the sample data is somewhat skewed, with an outlier, but she will proceed with some reservations.

(c) The boxplots show the populations are highly skewed, so she shouldn't proceed.

(d) The boxplots show the sample data is perfectly symmetric, with no skewing or outliers, so she can proceed.

(e) The boxplots can't tell her anything about the population distributions.

3 H. Use the given F-Table to answer the following questions

Source	DF	SS	MS	F-Stat	P-value
Columns	2	47.266667	23.633333	5.6519043	0.0089
Error	27	112.9	4.1814815		
Total	29	160.16667			

(a) What number tells you the mean variance BETWEEN the groups? $MS_{factor} = 23.633$

(b) What number tells you the mean variance WITHIN the groups? $MS_{error} = 4.181$

(c) How is the F-Stat found, using the numbers in the table? (Show the calculation and verify it matches the F-Stat in the table.)

Calculation of F-stat: $F = \dfrac{MS_{factor}}{MS_{column}} = \dfrac{23.633}{4.181} = 5.652 = $ F-Stat

1 I. In general (NOT about this specific study) if the F-stat in an ANOVA test is quite small, what can you tell? (Circle the best answer.)

(a) The signal (variance between groups) is weak relative to the noise (variance within the groups)
(b) The P-value will be large.
(c) The test won't show a significant difference between the groups.
(d) All of the above.
(e) None of the above.

4 J. Determine the conclusion of the hypothesis test for the Sound Study. The F-table is recopied here for convenience.

Source	DF	SS	MS	F-Stat	P-value
Columns	2	47.266667	23.633333	5.6519043	0.0089
Error	27	112.9	4.1814815		
Total	29	160.16667			

Conclusion (interpret results of hypothesis test):

P-value $= .0089 < .05 = \alpha$
Reject Ho, accept Ha
The data provides strong evidence that at least one of the groups had, on average, significantly different test scores than the other groups

Can Maddie conclude that sound during studying CAUSES a difference in how well students will do on a test, on average? Briefly explain.

Yes, since this is a controlled experiment she can conclude a cause-and-effect relationship.