

**Math 247: What Is Randomness?** (Section 5.1) and **Finding Empirical Probabilities** (Section 5.4)

**Probability Experiment:** In **probability** theory, an “experiment” or “**trial**” is a procedure that has a well-defined set of possible outcomes, known as the sample space.

**Sample Space:** All the possible outcomes of a probability experiment.

**Event:** A specific type of outcome.

**Probability of an Event:** If we use a letter to denote an event (e.g. A = pick a red card from a standard deck of cards), then we can define what the probability of an event is as follows:

The probability that event A occurs is the number of outcomes that match the event divided by the total number of possible (equally likely) outcomes.

Notation: 
$$P(A) = \frac{\text{number of outcomes that match the event}}{\text{total number of equally likely outcomes}}$$

The value of a probability can be between 0 and 1, expressed as  $0 \leq P(A) \leq 1$

- $P(A) = \underline{\hspace{2cm}}$  means the event cannot occur.
- $P(A) = \underline{\hspace{2cm}}$  means the event is CERTAIN to occur.

Any percentage (proportion) that is derived from a data set can be thought of as a probability.

**Example:** If you roll a six-sided die once, what is the probability it will come up as an even number?

What is the Sample Space?

What is the Event?

How many outcomes match the event?

What is the probability?

**Example:** If you were to draw a marble from a bag with 2 red, 3 blue, and 5 yellow. What is the probability the marble will be blue?

What is the Sample Space?

What is the Event?

How many outcomes match the event?

What is the probability?

**Theoretical Probability:** *a priori* You don't perform the experiment, but reason out how many outcomes match the event and how many outcomes there are, theoretically.

**Example:** What is the probability of getting a head if you flip a fair coin once?

Use proper notation!

**Empirical Probability:** *post priori* You actually perform the experiment (like flipping a coin), then count up the number of outcomes that match the event.

**Example:** Suppose you flip a fair coin 1000 times (you have a lot of time on your hands!) and it comes up heads 517 times. What is the empirical probability of getting heads with that coin?

Use proper notation!

### How are theoretical and empirical probabilities related?

**The Law of Large Numbers:** If you perform a large number of trials of a probability experiment, the empirical probability will be very close to the theoretical probability.

How we write this in a very "mathy" way:

$$\lim_{n \rightarrow \infty} P_{EMPIRICAL} = P_{THEORETICAL}$$

(where n = the number of trials)

