**Math 247: Comparing Proportions from Two Populations** (Sections 7.5, 8.4)

Does banding penguins in order to study and track them actually cause harm to the penguins? Claire Saraux, a wildlife researcher, conducted an experiment to answer this question. In 1998, she tagged 200 king penguins at Possession Island in Antarctica. She fitted half the birds with steel bands, and the other half with internal electronic tags. After 10 years of monitoring, Saraux found that just 20 of banded birds had survived the decade, compared to 36 of electronically tagged ones.

What is the research question for this study?

What are the two "populations" of interest here?

What are the two samples?

What is a "success" in this context?

We could organize Claire's data in a table:

| | Metal Band | No Metal Band |
|---|---|---|
| Survived (success/event) | | |
| Didn't survive (failure) | | |
| Total | | |

What **proportion** of the banded penguins survived? Use proper notation!

What **proportion** of the unbanded penguins survived? Use proper notation!

What is the difference between the survival **proportions**? Use proper notation! ________________________

What does the negative sign tell you?

Can we infer from this result that IF half of ALL penguins were banded and the other half of ALL penguins were not banded, that there would be EXACTLY a 16% difference in survival rates? Explain.

What should the **difference** be between survival *proportions* if the bands don't affect the penguins' survival?

Difference between the *populations*:

Difference between the *samples*:

Just as we found for a single proportion, if certain conditions are met, then the **differences** between the SAMPLE proportions will be approximately _______________________________ in their distribution.

**Conditions**:

1. **Random and Independent.** Each sample is randomly drawn from its population, and observations are independent from one another.

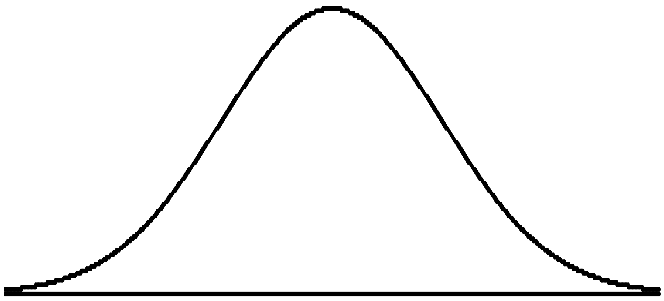
2. **Large Samples**. Both sample sizes are large enough that there are at least 10 expected successes and at least 10 expected failures in each sample. For this step, we use the "Pooled Proportion" $\hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$ which gives the overall proportion of successes in both samples combined.

3. **Large Populations**. The population size must be at least 10 times sample size, for each sample.

4. (New condition!) **Independent Samples**. The samples themselves are independent from each other.

   For example, if the banded birds and the unbanded birds were deliberately chosen as siblings, then the two samples would not be independent from each other!

Sampling distribution for the difference between two proportions:



Pooled Standard Error:

$$SE = \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$$

Unpooled Standard Error:

$$SE = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1}+\frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

Now that we have a sampling distribution, we can make an inference about the difference between the population proportions based on the difference between our sample proportions. In laymen's terms, we can see if the data provides evidence that there would be a significant difference in survival rates between banded and unbanded penguins for ALL penguins (not just the penguins in the samples!).

**Hypothesis Test for Two Proportions**

$H_0$ : The proportion of successes in each of the two populations is the same.

$p_1 = p_2 \Rightarrow \quad p_1 - p_2 = 0$ (the hypothesized difference is zero)

$H_a$ : The proportion of successes of the first **population** is less than the proportion of the second **population**

$p_1 < p_2 \quad \Rightarrow \quad p_1 - p_2 < 0$

**or** the first proportion is more than the second proportion

$p_1 > p_2 \quad \Rightarrow \quad p_1 - p_2 > 0$

**or** the first proportion different from the second proportion

$p_1 \neq p_2 \Rightarrow \quad p_1 - p_2 \neq 0$

We'll use StatCrunch to do the calculations but for reference, here are the formulas:

Test statistic: $z = \dfrac{\hat{p}_1 - \hat{p}_2}{SE}$ where $\hat{p} = \dfrac{x_1 + x_2}{n_1 + n_2}$ is the "pooled proportion" and

$SE = \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$ is the "Pooled Standard Error"

Test whether the penguin data suggests there is a statistically significant **difference** in penguin survival rates based on banding. For the hypothesis test, do Step 1 by hand, do Step 2, do Step 3 by hand then using StatCrunch, and do Step 4 by writing a complete conclusion that answers the research question.

**Step 1: ______________________________**

**Step 2: _______________________________**

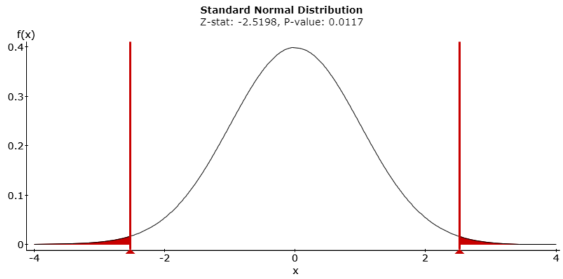
**Step 3:** ______________________________

**Using StatCrunch:** More complete instructions are at the end of Chapter 8

Stat → →Proportion Stats →Two Sample →With Summary
Fill in the number of successes and the number of observations for each sample
Select the "P-value plot" under "Optional Graphs and Tables"

**Hypothesis test results:**

| Difference | Count1 | Total1 | Count2 | Total2 | Sample Diff. | Std. Err. | Z-Stat | P-value |
|---|---|---|---|---|---|---|---|---|
| $p_1 - p_2$ | 20 | 100 | 36 | 100 | -0.16 | 0.0634 | -2.51976 | 0.0117 |




**Step 4:** ______________________________