Math 247: Comparing Two Population Means (Section 9.5)

Dependent Samples vs. Independent Samples

Suppose you're going to do your Senior Project on using meditation to reduce stress. How would you do this?

Using the same group of people	Using two different groups of people
Paired t-Test: Used for these **Dependent** samples: • Before-and-After studies • Twin studies • Any situation where there is a natural pairing of the data values.	**Two Sample t-Test:** Used for **Independent** samples This is the most common situation in experimental studies where there is random assignment to two groups.
 	 

Dependent Samples: Paired t-Test

Step 1: _____

State hypotheses using math symbols. Then describe what those symbols are saying, in words.

$H_0 : \mu_D = 0$

$H_a : \mu_D < 0$ (Left-Tailed) or $\mu_D > 0$ (Right-Tailed) or $\mu_D \neq 0$ (Two-Tailed)

Step 2: _____

Choose and **state** which test you're using and choose the Significance Level, α (alpha)

Check Conditions (make assumptions) FOR THAT TEST!

For the **Paired t-Test,** we have to meet these conditions**:**

Conditions:

1. **Random Sample and Independence within sample.**
2. **DEPENDENCE between samples.** The two samples themselves are DEPENDENT from each other. Subjects in each group are pair-matched (before-and-after, twin studies, couple studies, etc.)
3. **Large Sample OR Normal Population.** Either the sample size is 25 or more ("large" samples) OR if, n < 25, ("small" sample) the underlying population is approximately normal.

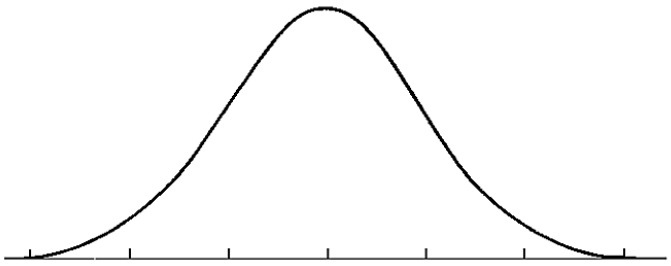
Step 3: _____

Find the Test Statistic by hand, but then we'll use StatCrunch to find the P-value. Be able to illustrate all of this on a t-distribution curve.

$$SE = \frac{s_D}{\sqrt{n}}$$

$$t_D = \frac{\bar{D} - \mu_D}{SE} = \frac{\bar{D}}{SE}$$

$$df = n - 1$$



Step 4: _____

Include both of the following:

(a) Compare the p-value to the level of significance and state whether you will reject or not reject the null hypothesis

(b) Interpret the result in the context of the problem. Your interpretation should include whether or not your result is statistically significant.

Meditation and Resting Heart Rate: Suppose a random sample of 10 people is drawn to study the link between meditation and resting heart rate. The entire group is assigned meditation for 20 minutes each day over a 6-week period. The raw data is given below.

Subject	1	2	3	4	5	6	7	8	9	10
Before (bpm)	74	67	83	70	77	83	70	75	72	88
After (bpm)	70	67	72	65	78	75	60	70	64	86
D = Difference										

Find the difference in heart rate for each subject.

If meditation and heart rate are unrelated, we'd expect the difference in heart rate to be _____, on average.

Looking at this data, does it appear that meditation actually does link to a change in heart rate?

How can we tell if the change is statistically ***significant***? _____

Step 1: _____

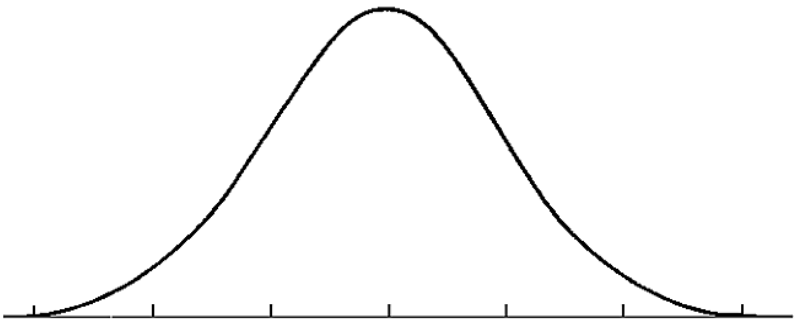
Step 2: _____

Step 3: _____

Do this step by hand, then use StatCrunch to find the P-value.

Summary statistics (for the Difference in heart rate)

Column	n	Mean	Std. dev.
Difference	10	5.2	4.077036



Paired T hypothesis test:

$\mu_D = \mu_1 - \mu_2$: Mean of the differences between Heartrate Before and Heartrate After

$H_0 : \mu_D = 0$ $H_A : \mu_D \neq 0$

Hypothesis test results:

Difference	Mean	Std. Err.	DF	T-Stat	P-value
Heartrate Before - Heartrate After	5.2	1.289272	9	4.033284	0.003

Step 4:

Follow-up: Can we say mediation CAUSED this significant change in heart rate? Why or why not?

Confidence Intervals for Differences Between Groups:

Confidence Interval Format: Estimated Difference ± Margin of Error

$$\bar{D} \pm t^* \cdot \frac{s_D}{\sqrt{n}}$$

Example: Find the confidence interval for the paired difference between the before and after groups. Then use StatCrunch to confirm your results. (Note: You'll need to use the t-table to find t*.)

Paired T confidence interval:

$\mu_D = \mu_1 - \mu_2$: Mean of the difference between Heartrate Before and Heartrate After

95% confidence interval results:

Difference	Mean	Std. Err.	DF	L. Limit	U. Limit
Heartrate Before - Heartrate After	5.2	1.289272	9	2.2834642	8.1165358

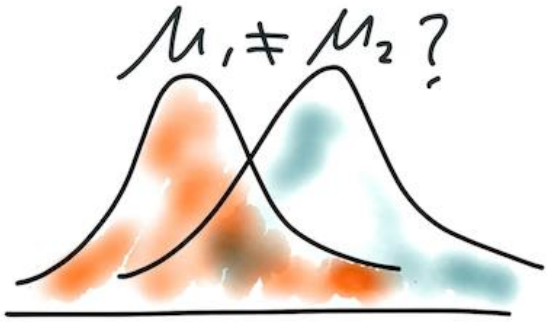
Graph and Interpret CI:

Section 9.5 (continued)

Two Sample t-test for Means (unpooled variance)

Independent Samples. If our two samples are not linked (i.e., they're not paired or dependent in some other way), then we use a Two Sample t-Test for Means

In comparing two populations, remember that individual values will vary and there may be overlap between the populations, but ON AVERAGE there can still be a significant difference between the populations.



Consider heights of women and heights of men:

Are some individual women taller than some individual men?

On average, are men taller than women?

1. Hypothesize:

$H_0 : \mu_1 = \mu_2 \Rightarrow \mu_1 - \mu_2 = 0$ There is ZERO difference, on average, between the two populations with regard to the variable of interest.

$H_a : \mu_1 \neq \mu_2 \Rightarrow \mu_1 - \mu_2 \neq 0$ There IS a difference, on average, between the two populations with regard to the variable of interest.

(Note: The alternative hypothesis above is for a _____-Tailed Test. We could also use a One-tailed test, but then we couldn't compare the results with the Confidence Interval results.)

2. Plan and Prepare:

Set significance level, alpha.

Choose Test: **Two Sample t-test for Means (unpooled variance)**

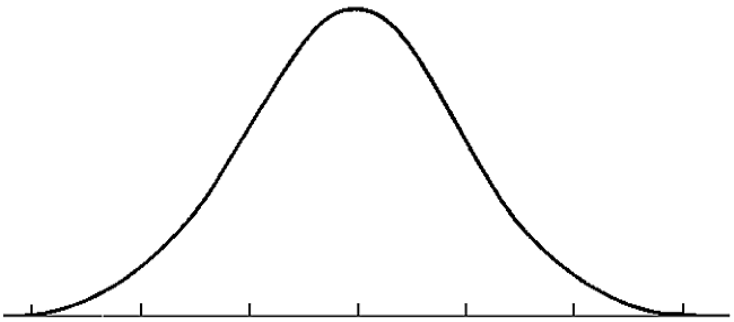
Conditions:

1. **Random Samples and Independence within samples.** We have two random samples from the two populations. Each observation in each sample is independent from all others.
2. **Independence between samples.** The two samples themselves are independent from each other. The individuals in one sample are in no way associated with the individuals in the other sample.
3. **Large Samples or Normal Pops.** Either the sample size in each sample is 25 or more ("large" samples) OR if, $n < 25$, ("small" samples) the underlying populations are each approximately normal.

Note: As with the previous Central Limit Theorems, if the conditions above are met, then the Sampling Distribution of the differences between sample means will approximately follow the t-distribution*

3. Compute

Find the probability that we'd get the observed value (difference in the sample means) IF there truly was ZERO difference in the population means. This probability is the P-value!



- Sketch sampling distribution of $\bar{x}_1 - \bar{x}_2$, centered on the Null ($\mu_1 - \mu_2 = 0$)
- Plot the observed difference in sample means. (The "point estimate")
- Calculate the Standard Error (unpooled): $SE = \sqrt{\frac{{s_1}^2}{n_1} + \frac{{s_2}^2}{n_2}}$
- Calculate the Test statistic: $t = \frac{\bar{x}_1 - \bar{x}_2 - 0}{SE}$
- You don't have to calculate the Degrees of Freedom, by hand, but here's the CRAZY formula we won't use!

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{s_2^2}{n_2}\right)^2}$$

  By hand, we could estimate degrees of freedom as

  df = least of $n_1 - 1$ and $n_2 - 1$

  (This is just for reference…StatCrunch will do the compute step for us, using the more complex, and more accurate formula.)

- Use StatCrunch to find the P-value and confirm your "by hand" calculations.

4. Interpret

(a) Compare the p-value to the level of significance and state whether you will reject or not reject the null hypothesis

(b) Interpret the result in the context of the problem. Your interpretation should include whether or not your result is statistically significant.

Example: Handedness and typing speed. Is handedness associated with how fast people type? Two random samples of left-handed students and of right-handed students who completed an online typing class are given a typing speed test (words per minute), and the results are compared. The 16 students in the right-handed sample had an average typing speed of 55.8 words per minute (wpm) with standard deviation of 5.7 wpm. The 9 students in the sample of left-handed students had an average speed of 59.3 wpm with a standard deviation of 4.3 wpm.

List and label the given values (Parking Lot!)

Is this a balanced design or an unbalanced design?

Looking at the sample data, is there a difference in how fast people type, on average, based on whether they're left-handed or right-handed? What is the "point estimate" of this difference?

How can we determine whether difference in typing speeds in the samples shows there's a significant difference in the populations of all right-handed and all left-handed people who type?

Conduct a hypothesis test to determine if there is a significant difference in typing speed in right-handed and left-handed people who take a typing class. Use a .05 level of significance.

1.

2.

3.
(Do work by hand then use StatCrunch to check and to get the P-value.)

StatCrunch results.

Hypothesis test results:

Difference	Sample Diff.	Std. Err.	DF	T-Stat	P-value
$\mu_1 - \mu_2$	-3.5	2.0211555	20.795052	-1.7316827	0.0981

4.

Confidence Intervals for the Difference in Means, Independent Samples:

Review: What is the relationship between confidence intervals and hypothesis tests?

- If a confidence interval captures the null, then this means the null could be _____ so we will not reject it.

- If the confidence interval does NOT capture the null, then this is evidence that the null is most likely

_____, so we WILL reject it.

What is the null when you're comparing two means? _____.

Summary: When looking a confidence interval for the difference in two means (also two proportions)

- If the confidence interval captures ZERO, there is NOT a significant difference in population means.
- If the confidence interval does NOT capture zero, then there IS a significant difference in population means.

Find (use StatCrunch) and interpret the confidence interval for the typing example above:

Two sample T summary confidence interval:

μ_1 : Mean of Population 1
μ_2 : Mean of Population 2
$\mu_1 - \mu_2$: Difference between two means
(without pooled variances)

95% confidence interval results:

Difference	Sample Diff.	Std. Err.	DF	L. Limit	U. Limit
$\mu_1 - \mu_2$	-3.5	2.0211555	20.795052	-7.7057467	0.70574671