Math 247: Invisible Errors: Type I and Type II Errors in Hypothesis Testing (Section 8.3)

Suppose we've done everything correctly in gathering our data, doing our analysis via hypothesis testing, then forming a conclusion based on the P-value.

There is still the possibility, due to _____, that the evidence led us to a conclusion that is incorrect!

Type I Error (the book calls this "the first type of error") when the evidence tells us to reject the null hypothesis when, unbeknownst to us, the null hypothesis is actually true.

The significance level, alpha, sets the maximum risk of making this error. The P-value further refines this and tells us precisely what the chance is we've made this error, if we do reject H_0.

$$\alpha = P(Type\ I\ error)$$

Type II Error (the book calls this "the second type of error"):

We make this error (and will never know we did!) when the evidence tells us to NOT reject the null hypothesis when, unbeknownst to us, the null hypothesis is actually false.

We make this error if we decide to not reject the null hypothesis, when the null hypothesis actually was false.

The likelihood of this type of error (failing to reject the null hypothesis when it really wasn't true) goes up as the significance level gets smaller.

$\beta = P(Type\ II\ error)$. Calculating this value is much more complex than finding a P-value, so we won't do it in this course.

Type I and Type II Error Grid		**Reality** (what's actually true of the **population**)	
		***H*₀ is True** ***Ha* is False**	***H*₀ is False** ***Ha* is True**
Conclusion of Hypothesis Test (based on the evidence given by the **sample data**)	**Reject *H*₀ and accept *Ha*** **POSITIVE result**	**Type I error** **False Positive** $\alpha = P(Type\ I\ error)$	Correct
	Don't reject *H*₀ **NEGATIVE result**	Correct	**Type II error** **False Negative** $\beta = P(Type\ II\ error)$

Example 1: Right now, there is a lot of research going on to determine whether the hydroxychloroquine is helpful in treating COVID-19 (the disease caused by the coronavirus). Researchers in Wuhan did a controlled, randomized experiment with 62 COVID-19 patients to determine whether the proportion of improved* patients in the treatment group was higher than the proportion of improved* patients in the control group.
(*Specifically, they were looking at pneumonia outcomes in the patients.)

Hypotheses:

H_0:

H_a:

Describe what a Type I error would be in this context:

Describe what a Type II error would be in this context:

The P-value from the analysis of pneumonia outcomes was .0176. What is our conclusion, based on this P-value?

What error could we be making here? False positive = Type I Error False negative = Type II Error

What harm could this error cause?

Example 2: Another drug under study right now, lopinavir–ritonavir, is one that has been used successfully for HIV treatment. The researchers in Wuhan used a randomized, controlled trial to see whether this drug, which looked promising in vitro (meaning in a lab setting), would actually be effective in vivo (meaning with actual patients). They were measuring, among other things, the percentage of patients who had improved on each day in the treatment vs. control groups to see whether there was a significant difference.

Hypotheses:

H_0:

H_a:

Describe what a Type I error would be in this context:

Describe what a Type II error would be in this context:

If the P-value was .12, what conclusion would the researchers make?

Suppose in reality, if the drug were given to the entire population of patients with severe COVID-19, there WOULD be an improvement by day 10. By failing to reject the null, what type of (invisible) error would we have made?

False positive = Type I Error False negative = Type II Error

What harm could this error cause?