Math 247: Continuous Random Variables: The Normal Distribution (Section 6.2)

## Distributions of Continuous Random Variables:

$\mathrm{X}=$ height is a continuous R.V. Suppose we had data on all the heights of all the men and women in the US.

Heights of people in the U.S., (assume entire population)

The distribution might not be perfectly normal, but it would be close enough that we could start saying some things about heights, as in what's an average height, what's pretty common and what's unusual.


Estimate the mean and standard deviation for each distribution. Comment on the spread and its relationship with the standard deviation. (Assume each curve represent the entire population and use proper notation.)


What are common heights for U.S. women, based on the information above?

If we know the mean and standard deviation we've see (in Section 3.2) that we can use z-scores and the Empirical Rule to judge what's common or unusual for a normally distributed Random Variable (like height).

The z-scores create their own normal curve, called the "Standard Normal Curve".

The Normal Distribution: The $N(\mu, \sigma)$ curve. $\mu=$ population mean, $\quad \sigma=$ population standard deviation


The Standard Normal Distribution: $N(0,1)$ curve This curve is the "skeleton" underneath every normal distribution curve. $z=\frac{x-\mu}{\sigma}$


The Empirical Rule, applied to the Standard Normal (z) Distribution looks like this:


Example: Write each problem using the proper notation. Then find the indicated probability using the Empirical Rule.

What's the probability of z being -2 or less?

What's the probability of $z$ being 3 or more?

What's the probability of z being between -1 and 1 ?

What if our z -values aren't nice, perfect whole numbers? To find probabilities, we'll have to use technology.
***Stop here and watch the video on how to use the StatCrunch Normal Distribution Calculator***
Example: Write each problem using the proper notation. Then find the indicated probability using the StatCrunch Normal Distribution Calculator.

What's the probability of $z$ being -1.73 or less?

What's the probability of $z$ being 2.18 or more?

What's the probability of z being between -.93 and .93 ?

Example: Heights of U.S. Women. The mean height for U.S. women is 65 inches, with a standard deviation of 2.5 inches.

Fill in the values of the heights on the z -score graph.
z-scores


Heights

Find the percentage of women who are between 62.5 in and 67.5 inches
(a) According to the Empirical Rule
(b) Using the Normal Distribution calculator on StatCrunch:

Note: Use the z -scores!

Options $\quad \approx x$


Example: Heights of U.S. women (continued)
Is a height of $5^{\prime} 9^{\prime \prime}=69$ inches unusual for a woman? (Mean height is 65 in, S.D. is 2.5 in )
Answer this question by finding the $z$-score for this height.

Now, try find the probability that a woman is exactly 69 inches tall using StatCrunch.
Use the "Between" command and put the same z -score in both boxes.

Options $88 \times$


## Finding probabilities using the Normal Distribution.

Example: For a certain type of computers, the length of time between charges of the battery is normally distributed with a mean of 50 hours and a standard deviation of 6 hours.

Find the probability that the length of time between charges will be
(a) less than 40 hours.
(b) more than 62 hours.
(c) between 45 and 50 hours

Inverse Normal Distribution: What if we know a percent (or probability) and want to find the x -value that goes with it? In order to that we have to use the Inverse Normal Distribution.

Percentile: The kth percentile of a data set is the x -value that has k percent of the data below it.
Example: The average birthweight of baby elephants is 230 pounds (wow!). Assume the distribution of birth weights follows the normal distribution $\mathrm{N}(230,50)$.
(a) Sketch a well-labeled curve for the distribution of baby elephant birth weights.

(b) Use the StatCrunch Normal Distribution Calculator to find the $25^{\text {th }}$ percentile for baby elephant birth weights.

Steps:

1. Sketch a Normal Curve with the indicated percentage shaded in.
2. Pull up the Normal Distribution

Calculator on StatCrunch
3. Fill in the mean and S.D.
3. Choose the $\leq$ symbol
4. Fill in the percentage in the probability box.
5. Click COMPUTE.


What is another name for the $25^{\text {th }}$ percentile? $\qquad$

