Math 247: Test 2 **Name:** KEY

In-class test ________ 70 ~~/80~~ *points* *Take home test* ________ 30 ~~/20~~ *points*

On all problems involving probability, express your answer as a fraction, decimal, and percent.

1. (2 pts) Which of the following numbers could NOT be probabilities? Circle your answer(s).

a) 0.724 b) 1.043 c) 0.125 d) -0.432 e) 1

2. (a) What is the probability of getting heads, if you flip a fair coin once? Express your answer as a fraction, decimal, and percent.
(9pts)

$P(\text{head}) = \frac{1}{2} = .50 = 50\%$

(b) If you flipped a fair coin 1000 times, how many heads would you <u>expect</u> to get?

$E = np = 1000(\frac{1}{2}) = 500$ heads

(c) Suppose you actually DID flip a coin 1000 times and got heads 512 times. What is the probability of getting heads based on this actual experiment?

$P(\text{heads}) = \frac{512}{1000} = .512 = 51.2\%$

(d) Explain why the answers in (a) and (c) aren't the same and include what you would have to do, in terms of coin flipping, to have the <u>actual</u> coin flip probability get closer to the answer in part (a).

The probability that's theoretical (50%) will generally never be matched by the empirical result (51.2%) due to sampling variability. If we continued flipping, say, 1 million times, we would see the empirical probability get very close to 50%.

(e) What is the name of the Law that relates the actual coin flip probabilities to the theoretical value?

The Law of Large Numbers

3. (~~5~~ 4 pts) Use your knowledge of the world to determine whether the following events are independent:

A = a person is tall B = a person plays basketball C = a person drives a white car

associated

A and B are independent — False

A and C are independent — True

B and C are independent — True

4. (~~4~~ 2 pts) Suppose Event A is that a person is sleeping. Give an example of another event, Event B, that is mutually exclusive to Event A.

Event B = answers will vary

Examples: B = not sleeping OR B = eating, etc

"running a marathon" "swimming" "shopping" "performing brain surgery" "climbing a tree"

(2)

5. (a) If Event A = Rain today, what is the complement of A? $A^C =$ no rain today

(2)

(b) If P(A) = .80, what is $P(A^C)$? $P(A^C) =$ .20

6. Some dice are 8- sided. If you rolled an 8-sided die once and flipped a coin,
(3) what is the probability you would get a five on the die OR tails on the coin?

P(five OR tails) = P(five) + P(tails) =

$= \frac{1}{8} + \frac{1}{2} = .625 = 62.5\%$

7. If you draw one card from a deck of playing cards
(3) (52 cards, 4 suits of hearts, spades, clubs, diamonds, 13 cards in each suit),
what is the probability you'll draw either a queen or a heart?

P(Q OR ♡) = P(Q) + P(♡) − P(Q and ♡)

$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = .308 = 30.8\%$

8. (3 pts) The Humane Society of the United States reported that 25% of households own a dog and 42% own a cat. Would
(1) it be correct to say that 67% of households own a dog **or** a cat? YES, correct NO, not correct

(2) Explain your answer. Include the "OR" formula as part of your answer.

P(dog OR cat) = P(dog) + P(cat) − P(dog AND cat)

$= .25 + .42 - ?$

Some households will own both a dog AND a cat and so will be double counted if we just had P(dog) and P(cat) together.

9. (~~3~~ pts) Suppose a bag contains 5 red marbles and 15 yellow ones.

Keep at 3pts

(a) If you select one marble,
what is the probabilitiy of getting a red one?

$P(\text{red}) = \frac{5}{20} = .25 = 25\%$

(b) If you sample two marbles, with replacement,
what is the probability of getting two red marbles?

$P(\text{red 1st and red 2nd}) = \frac{5}{20} \cdot \frac{5}{20} = .063 = 6.3\%$

(c) If you sample two marbles, without replacement,
what is the probability of getting two red marbles?

$P(\text{red 1st and red 2nd}) = \frac{5}{20} \cdot \frac{4}{19} = .053 = 5.3\%$

10. (~~15~~ 7 pts) A survey of randomly selected adults found that 11% of the men said they thought adult children shouldn't live with their parents. Let N = no kids at home, and K = kids okay at home. $P(N) = .11 \quad P(K) = .89$

(a) If two men meet, list all of the possible outcomes of their opinions. Use N and K for this.

$\boxed{KK \quad NK \quad KN \quad NN}$

(b) What is the probability that both of them will have the opinion that it's okay to have adult children live with their parents?

$$P(KK) = P(K) \cdot P(K) = (.89)(.89) = .792 = \boxed{79.2\%}$$

(c) If two men meet, what is the probability they will disagree on this issue.

$$P(NK \text{ OR } KN) = P(NK) + P(KN) = (.11)(.89) + (.89)(.11)$$
$$= .196 = \boxed{19.6\%}$$

(d) To find the values in parts (b) and (c), you have to assume the opinions of the two men are independent. If the men were allowed to discuss the issue first, THEN were asked whether or not adult children are okay at home, would their answers still be independent? Briefly explain.

No, since one man's opinion might affect the other man's, their answers would no longer be independent.

11. (~~20~~ 10 pts) The given table shows the data relating sex and handedness for 121 students. Use the table to answer the following questions:

	Handed Left	Handed Right	
Female	7	46	53
Male	5	63	68
	12	109	121

(a) What percentage of the students are left handed?

$$P(\text{Left}) = \frac{12}{121} = .099 = \boxed{9.9\%}$$

(b) What percentage of the female students are left handed?

$$P(\text{Left} \mid \text{Female}) = \frac{7}{53} = .132 = \boxed{13.2\%}$$

(c) Based on just your answers to (a) and (b), is there an association in this group of students between being left-handed and being female? Explain your answer.(Note: Do NOT do a Chi-Square Test here. Just examine your answers from (a) and (b).)

Yes, since $P(\text{Left}) \neq P(\text{Left} \mid \text{Female})$ we see that being female and being left-handed are related. A higher percentage of females are left-handed.

(d) What percentage of students are either male or right-handed?

$$P(\text{Male OR Right}) = P(\text{Male}) + P(\text{Right}) - P(\text{Male and Right})$$
$$= \frac{68}{121} + \frac{109}{121} - \frac{63}{121}$$
$$= .942 = \boxed{94.2\%}$$

12. **Vitamin C** A study (double-blind) was done investigating the therapeutic value of vitamin C (ascorbic acid) for treating common colds. The study (done in 1971 by Linus Pauling) was conducted during a 2-week period on a sample of 279 school children in a skiing camp in the Swiss Alps. The participants were split into two groups, one taking 1 gram of vitamin C per day and the other taking a placebo. At the end of two weeks the researchers assessed who had gotten a cold and who hadn't.

These are the data for this study:

		C	NC	
		Cold	**No Cold**	
P	**Placebo**	O = 31 E = 24.1	O = 109 E = 115.9	140
V	**Vitamin C**	O = 17 E = 23.9	O = 122 E = 115.1	139
		48	231	279

Work for Expected Counts:

$E_{P,C} = \frac{140 \times 48}{279} = 24.1$

$E_{P,NC} = \frac{140 \times 231}{279} = 115.9$

$E_{V,C} = \frac{139 \times 48}{279} = 23.9$

$E_{V,NC} = \frac{139 \times 231}{279} = 115.1$

Using the data in the table, do a Chi-Square Hypothesis Test to see whether taking Vitamin C is associated with prevention of colds. Use $\alpha = .05$

(4) (a) Hypothesize: Write the Null and Alternative Hypotheses

H_0: Vitamin C use and Cold Status are independent

H_a: Vitamin C use and Cold Status are associated

(6) (b) Prepare: Level of significance = .05 (1) Find and fill in the expected counts. Show work (3)

(2) What do you have to check regarding expected counts to make sure the Chi-Square Test is valid?

$E \geq 5$ (which all are!)

(4) (c) Find χ^2 and the degrees of freedom (DF) by hand. Show work!

$$\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(31-24.1)^2}{24.1} + \frac{(109-115.9)^2}{115.9} + \frac{(17-23.9)^2}{23.9} + \frac{(122-115.1)^2}{115.1}$$

$DF = (2-1)(2-1) = 1$ $\chi^2 = 4.811$

$\chi^2 =$ 4.811 DF = 1

(5) (d) Interpret: The Minitab result from a Chi-Square Association Test is given to the right. Use this to write the conclusion to the hypothesis test.

	Chi-Square	DF	P-Value
Pearson	4.811	1	0.028

Conclusion: There is a 2.8% chance the apparent association is just due to sampling variability. We have sufficient evidence, therefore, to $\boxed{\text{reject } H_0}$ and conclude the is a significant association between Vitamin C use and getting or not getting a cold.

(e) Does this study show that watching taking vitamin C can *cause* a reduction in colds? $\boxed{\text{Yes}}$ No

yes, experimental design, but χ^2 doesn't show reduction, only difference!