Math 247: Test 2 **Name:** KEY

_____/100 points

On all problems involving probability, use the correct notation for full credit on the problem.

1. (4 pts) Which of the following numbers could NOT be probabilities? Circle your answer(s).

(a) 2.724 b) 0.043 (c) −0.125 d) 0 e) 1

over 1 - no! negative - no!

2. (6 pts) Which is an Empirical Probability and which is a Theoretical Probability? Circle your answer.

a. You flip a fair coin 100 times and get heads 53 times. You calculate $P(Heads) = \frac{53}{100} = .53$

(Empirical) Theoretical You actually did the experiment and had data

b. You calculate the probability of getting a heart in a deck of cards as $P(Heart) = \frac{13}{52} = .25$

Empirical (Theoretical) You never touched an actual card, rather just thought about it.

3. (6 pts) If you draw one card from a deck of cards, which of the following outcomes are mutually exclusive?

A = Getting a king.
B = Getting a queen.
C = Getting a heart.

A and B are mutually exclusive? (YES) NO – no overlap

A and C are mutually exclusive? YES (NO) King of ♡'s = overlap

B and C are mutually exclusive? YES (NO) Queen of ♡'s = overlap

4. (4 pts) Use your knowledge of the world to label the pairs of these events as independent or associated.

(a) The outcome of each flip (H or T) when you flip a coin a coin twice independent

(b) Playing water polo and being a good swimmer associated

5. (5 pts) If you flip a fair coin and roll a six-sided die, what is the probability you will get Tails on the coin and a 5 on the die?

$P(\text{Tails AND } \boxed{\cdot\cdot}) = P(T) \cdot P(5) = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12} = .083 = 8.3\%$

Total of 15 marbles

6. (14 pts) Suppose you have a bag with 3 yellow marbles, 4 red marbles, and 8 blue marbles. Find the following probabilities and express each as a fraction, a decimal, and a percent.

(a) If you choose one marble,

a. what is the probability it will be red?

$$P(\text{red}) = \frac{4}{15} = .267 = 26.7\%$$

b. What is the probability it won't be red?

$$P(\text{red}^C) = 1 - P(\text{red}) = 1 - \frac{4}{15} = \frac{11}{15} = .733 = 73.3\%$$

c. What is the probability it will be red AND yellow?

$$P(\text{red AND yellow}) = \frac{0}{15} = 0 = 0\%$$

(no marbles are both red and yellow!)

d. What is the probability it will be red OR yellow?

$$P(\text{red OR yellow}) = P(\text{red}) + P(\text{yellow}) = \frac{4}{15} + \frac{3}{15} = \frac{7}{15} = .467 = 46.7\%$$

(b) If you choose two marbles **with replacement**, what is the probability both will be red?

$$P(2 \text{ red}) = P(\text{red AND red}) = P(\text{red}_1) \cdot P(\text{red}_2) = \frac{4}{15} \cdot \frac{4}{15} = \frac{16}{225} = .071 = 7.1\%$$

(c) If you choose two marbles **without replacement**, what is the probability both will be red?

$$P(2 \text{ red}) = P(\text{red and red}) = P(\text{red}_1) \cdot P(\text{red}_2 \mid \text{red}_1) = \frac{4}{15} \cdot \frac{3}{14} = \frac{12}{210} = .057 = 5.7\%$$

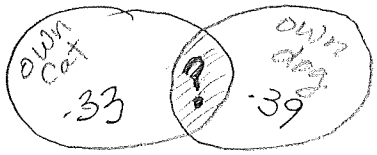
7. (5 pts) The Humane Society of the United States reported that 39% of households own dogs and 33% own cats.

Would it be correct to say that 72% of households own a dog or a cat? YES, correct NO, not correct

Explain your answer.

$P(\text{cat OR dog}) = P(\text{cat}) + P(\text{dog}) - P(\text{cat AND dog})$

The given percentages double count the households who own BOTH a cat and a dog.



? = percent who own BOTH.

8. (12 pts) If a woman has three separate children (no twins or triplets! ☹), what are the possible outcomes for the sex of her three children? (Assuming it's equally likely to have a boy or a girl and ignoring the possibility of an intersex child.) Use B for boy and G for girl.

Outcomes: BBB BBG BGB GBB BGG GBG GGB GGG

Fill in the table for the probability distribution of X = the number of girls in the set of 3 kids.

X	0	1	2	3
P(X)	1/8 = .125 = 12.5%	3/8 = .375 = 37.5%	3/8 = .375 = 37.5%	1/8 = .125 = 12.5%

Interpret the notation (write what it means, in words) and find the indicated probability:

$P(X = 0)$ = the probability there are no girls
= .125

$P(X \geq 1)$ = the probability there is at least 1 girl
= P(X=1 OR X=2 OR X=3)
= .375 + .375 + .125 = .875

EASIER: $P(X \geq 1) = 1 - P(X = 0)$
P(at least 1) = 1 − P(none) = 1 − .125 = .875

9. (10 pts) A survey of randomly selected adults found that 52% of the men and 68% of the women believe there is intelligent life on other planets.

a. If M = the event that a man believes, what is $P(M^C)$?

$P(M^C) = 1 - P(M)$
$= 1 - .52$
$= .48 = 48\%$

b. If a man and a woman from this group meet, what is the probability that the man does not believe AND the woman does believe?

$P(M^C \text{ AND } W)$
$= P(M^C) \cdot P(W)$
$= (.48)(.68) = .326 = 32.6\%$

10. (16 pts) Researchers asked students in grades 4 through 6 in three school districts in Michigan about what they thought was the most important thing in school: making good grades, being popular, or being good in sports. There were rural, suburban, and urban schools surveyed. The table shows the results

	Rural	Suburban	Urban	All
Grades	57	87	103	247
Popular	50	42	49	141
Sports	42	22	26	90
All	149	151	178	478

Express each of the following probabilities as a fraction, a decimal to three decimal places, and a percent.

a. What percentage of all the students think grades are the most important thing in school?

$\frac{247}{478} = .517 = 51.7\%$ of the students think grades are the most important.

b. What percentage of the Urban students think that grades are the most important thing in school?

$\frac{103}{178} = .579 = 57.9\%$ of the Urban students think grades are the most important.

c. Based on your answers to (a) and (b), is there an association between being from an Urban school, and thinking grades are most important for this group of kids? Explain your answer.
(Note: Do NOT do a Chi-Square Test here. Just explain by using your answers from (a) and (b).)

Since there is a higher percentage of Urban kids who think grades are most important, as compared to the entire group of all kids, there seems to be a slight association between Urban and grades.

d. What is the probability a student chosen from the entire group is either from a Suburban school or thinks that Popularity is most important?

$P(\text{Suburban OR Popular})$

$= P(\text{Sub}) + P(\text{Pop}) - P(\text{Sub AND Pop})$

$= \frac{151}{478} + \frac{141}{478} - \frac{42}{478} = \frac{250}{478} = .523 = 52.3\%$

11. (24 points) A study was conducted in 2013 to see whether it is better to give the diphtheria, tetanus and pertussis (DTaP) vaccine in the thigh for the arm. The researchers collected data on severe reactions to this vaccine in children aged 3 to 6 years old. The table below shows the data they used:

Location of Injection	No severe reaction	Severe reaction	
Thigh	O = 4758 E = 4750.96	O = 30 E = 37.04	4788
Arm	O = 8840 E = 8847.04	O = 76 E = 68.96	8916
	13,598	106	13,704

(note: All E's but first can be found using totals!)

Work for Expected Counts

Thigh Not Severe $E = \frac{4788 \times 13598}{13,704} = 4750.96$

Thigh Severe $E = \frac{4788 \times 106}{13704} = 37.04$

Arm Not severe $E = 13,598 - 4750.96 = 8847.04$

Arm Severe $E = 106 - 37.04 = 68.96$

Conduct all 4 steps of a Chi Square Test of Independence to see whether there is an association between the location of the injection and whether a child has a severe reaction to the vaccination. **Use a significance level of 0.05.**

Step 1: Hypothesize

H_0: There is no association between injection site and severe reactions.

H_a: There IS an association

Step 2: Prepare

Choose test: Chi Square Test for Independence

$\alpha = .05$

Check conditions:

1. Random sample from pop? No, but assume to continue.
   Independent observations? Assume
2. Large sample?
   $E \geq 5$ for all cells
   So yes!

Step 3: Compute

Find and fill in the expected counts on the table. Show work!

Find χ^2 and degrees of freedom by hand. Show work!

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

$$= \frac{(4758-4750.96)^2}{4750.96} + \frac{(30-37.04)^2}{37.04} + \frac{(8840-8847.04)^2}{8847.04} + \frac{(76-68.96)^2}{68.96}$$

$$= 2.073$$

$\chi^2 =$ 2.073 (small value due to small differences)

df = $(2-1)(2-1) = 1$

Step 4: Interpret (Make a Conclusion!)

The P-value for the Test is .1502. Use this information to complete the last step of the test. Be sure to write your final answer in the context of the problem.

P-value = .1502 > .05 = α

P-value is too large, so we fail to reject H_0.

There is NOT convincing evidence that there is any kind of link between the injection site (arm or thigh) and whether a child has a sorore reaction. It's more likely that site and reaction are independent.