Math 265A Chapter 1 Data Analysis:

Determining what type of function is (or could be) represented by a table of (x,y) values

Given the following table of function values for the same set of x-values,

- determine whether the function values represent a linear, quadratic, exponential or sinusoidal function
- find a formula that fits the data (for all but the quadratic function)
- determine the function value for x = 100

x	$f(x)$	$g(x)$	$h(x)$	$p(x)$
1	0	0.3	3.3	1.2
2	3	1.2	3.63	1.6
3	0	2.7	3.993	2.0
4	-3	4.8	4.392	2.4
⋮	⋮	⋮	⋮	⋮
100	?	?	?	?

<u>Linear</u>: *p* is the linear function. Why? Note that for this data the <u>rate of change</u> of the y-values (p(x)) is <u>constant</u>, i.e. $m = \Delta y / \Delta x = 0.4$ for all values in the table.

To find the formula, since we're looking for a line equation, use the point-slope formula:

$$y - y_1 = m(x - x_1)$$
$$y - 1.2 = 0.4(x - 1)$$
$$y = 0.4x + 0.8$$
$$p(x) = 0.4x + 0.8 \quad \Rightarrow \quad p(100) = 0.4(100) + 0.8 = 40.8$$

<u>Exponential</u>: *h* is the exponential function. Why? Note that the ratio of the successive y-values is approximately constant; specifically,

$$\frac{3.63}{3.3} = 1.1 \qquad \frac{3.993}{3.63} = 1.1 \qquad \frac{4.392}{3.993} \cong 1.1$$

(The ratio isn't exactly the same each time, but is close enough to being constant to ensure that using an exponential function to model this data is reasonable.)

To find the formula for the exponential function, $y = y_0 a^x$, we first use the fact that the base, a, is the constant ratio found above; i.e., a = 1.1. Then we'll substitute an (x,y) pair into the formula to solve for y_0 (Note: had we known the y-value for x=0, that would have been our initial y-value, y_0, and we'd be done with all work in finding the formula.)

Substitute a = 1.1 to get $y = y_0(1.1)^x$

Subtitute x=1, y = 3.3 to get $3.3 = y_0(1.1)^1$

Solve for y_0 to get $y_0 = 3$

So $h(x) = 3(1.1)^x \quad \Rightarrow \quad h(100) = 3(1.1)^{100} \cong 41342$

Quadratic: *g* is the quadratic function. Why? The identifying characteristic of quadratic functions is that the change in the CHANGE in the y-values is constant. We'll see why this is so in the next chapter. In the meantime, note that the data can be organized and analyzed as follows:

y	0.3		1.2		2.7		4.8
Δy		0.9		1.5		2.1	
$\Delta(\Delta y)$			0.6		0.6		

Since $\Delta(\Delta y)$ is constant, g is quadratic.

Sinusoidal: Of course, by the process of elimination, *f* must be sinusoidal, but here is how we could reason this out:

> The y-values of *f* are oscillating between 3 and -3 and are taking on the average of these values (y = 0) midway between these extremes which corresponds to the internal symmetry of a sinusoidal function graph. (Had x = 1 corresponded to y = 1 instead of y = 0, we'd be dealing with a different function, perhaps a polynomial.)

To find the formula, we can use the general sinusoidal function formula, i.e., $y = A\sin(B(x-h)) + C$

A = amplitude = 3 units, in this example.

h = the phase (horizontal) shift, which is 1 unit. Why? Loosely, the parent graph, y = sin(x) "begins" at the point (0,0). This graph "begins" at (1,0), hence is shifted right 1 units.

C = the vertical shift = the average value of the function = 0, in this example.

B is given by the formula $Period = \dfrac{2\pi}{B}$. The period of this function is, apparently, 4 units.

$$\text{So } 4 = \frac{2\pi}{B} \quad \Rightarrow \quad B = \frac{\pi}{2}$$

$$\text{So } f(x) = 3\sin\left(\frac{\pi}{2}(x-1)\right) \quad \Rightarrow \quad f(100) = 3\sin\left(\frac{\pi}{2}(100-1)\right)$$

$$= -3$$