

**Math 265A: Differentiability****Name:** \_\_\_\_\_

In this worksheet we will analyze the differentiability of a function graphically and analytically. You will need a separate sheet of paper.

I. First we will analyze the differentiability of  $f(x) = (x - 1)^{1/3}$

**A. Graphical analysis**

1. Sketch a graph of the function. Use a window of  $[-5, 5] \times [-5, 5]$

2. Visually estimate where there is a point ( $x = c$ ) where the derivative of  $f$  does not exist. Show this point on your graph and explain why the derivative does not exist there. Use a geometric argument, not one involving limits.

**B. Algebraic analysis**

**Using the specific point  $x = c$**  you observed in part A, do the following:

1. Write the limit definition (using difference quotients) of  $f'(c)$

2. Analyze the limit and explain why it fails to exist.

**C. Making a conclusion:**

$f$  is differentiable for all  $x$  values such that  $x \neq 1$

II. Repeat parts A and B for the function

$$g(x) = \begin{cases} 2x+1 & x \leq 2 \\ \frac{1}{2}x+4 & x > 2 \end{cases}$$

III. More practice: Analyze the differentiability of the following functions graphically and analytically.

1.  $f(x) = x^{\frac{2}{5}}$

2.  $f(x) = |x+2|$

3.

$$f(x) = \begin{cases} x-2 & x \leq 3 \\ -\frac{2}{3}x+3 & x > 3 \end{cases}$$