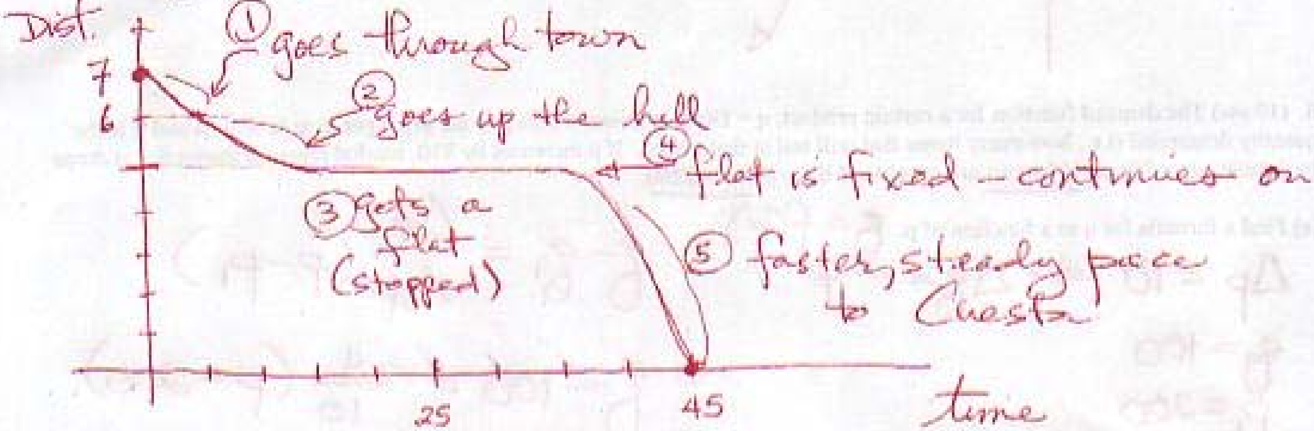
Math 265A: Test # 1 Name: KEY

For maximum credit, please organize all of your work in a clear, logical manner. Credit is based on amount of correct work you show, not on the answer alone.

1. (8 points) A student is traveling on her bike from San Luis Obispo to Cuesta, a distance of 7 miles. She pedals at a constant speed for the mile through town then slows down climbing the first hill, which is about a mile in length. At the top of the hill, she gets a flat, repairs it, then continues on at a steady but faster rate to make up for lost time. The entire trip takes her 45 minutes.

Draw graph that shows her distance FROM CUESTA with respect to time.

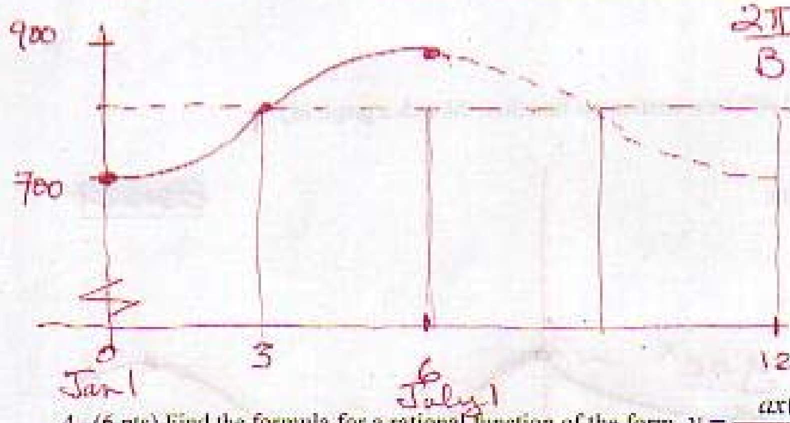
Include as many details as possible.



2. (5 pts) The value of a car, $V = f(a)$, in thousands of dollars, is a function of the age of the car, a, in years. Explain the meaning of $f(10) = 2$ in terms of the value of the car.

$10 = a$ = age of car
$2 = V$ = value of car
The car is worth \$2,000 when it's 10 years old

3. (8 pts) A population of animals oscillates sinusoidally between a low of 700 on January 1 and high of 900 on July 1. Find a formula for the population as a function of time, measured in months since the start of the year.



$\frac{2\pi}{B} = 12 \implies B = \frac{\pi}{6}$

$P(t) = 100\sin\left(\frac{\pi}{6}(t-3)\right) + 800$

OR

$P(t) = -100\cos\left(\frac{\pi}{6}t\right) + 800$

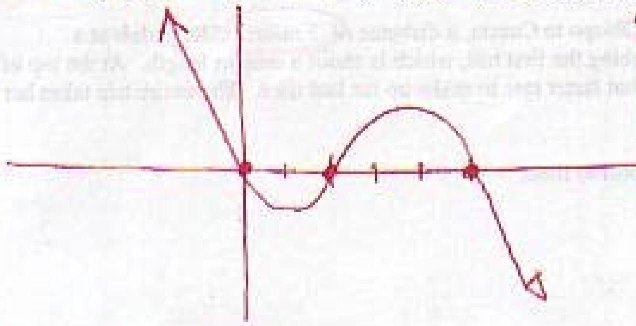
4. (6 pts) Find the formula for a rational function of the form $y = \dfrac{ax(x-c)}{(x-b)(x-c)}$ with a vertical asymptote of x = 1, a hole at x = 3 and a horizontal of y = 6.

$a = 6$
$c = 3$
$b = 1$

$$y = \frac{6x(x-3)}{(x-1)(x-3)}$$

Name KEY

5. (6 pts) Sketch a graph and find a formula for a polynomial function, $p(x)$, which has zeros of 2, 5 and 0 and with end behavior of $p(x) \to -\infty$ as $x \to \infty$; $p(x) \to \infty$ as $x \to -\infty$.



$$\boxed{p(x) = -x(x-2)(x-5)}$$

6. (10 pts) The demand function for a certain product, q = D(p), is linear, where p is the price per item in dollars and q is the quantity demanded (i.e., how many items that will sell at that price). If p increases by \$10, market research shows that q drops by 4 items. In addition 100 items are purchased if the price is \$200.

(a) Find a formula for q as a function of p. $q = f(p)$

$\Delta p = 10 \Rightarrow \Delta q = -4$

$q_1 = 100$

$p_1 = 200$

$$q - q_1 = \frac{\Delta q}{\Delta p}(p - p_1)$$

$$q - 100 = \frac{-4}{10}(p - 200)$$

$$\boxed{q = -\frac{2}{5}p + 180}$$

(b) Explain what the slope you found for part (a) tells you about price and sales of the item.

For every increase of \$5 the demand will decrease by 2 items.

7. (8 pts) Determine a value for k that will ensure that $f(x)$ will be a continuous function. Sketch a graph of f.

$$f(x) = \begin{cases} e^x & x < 0 \\ k\cos(x) & x > 0 \end{cases}$$

Graph:

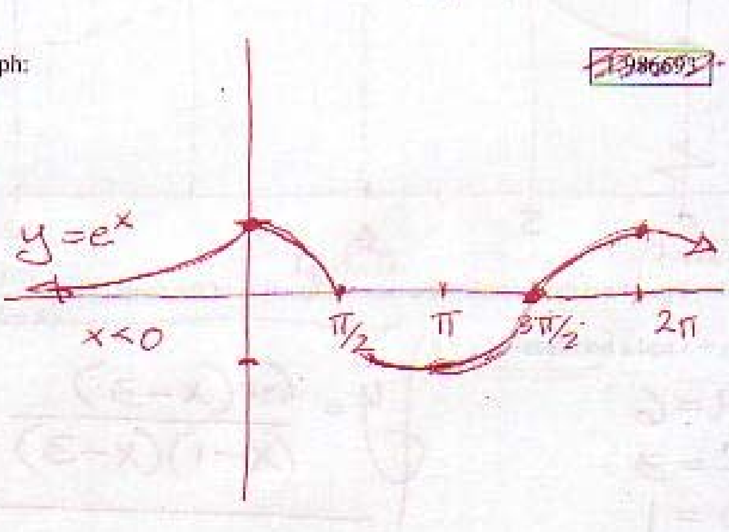
1986695

k = 1

$e^0 = k\cos(0)$

$1 = k(1)$

$k = 1$



8. (12 pts) The given table contains values for three different functions.

x	$f(x)$	$g(x)$	$h(x)$
-2	-5	5.1	28
-1	-2	10.71	24
0	0	22.49	20
1	2	47.23	16
2	5	99.18	12

(a) Which (if any) of these functions is a linear function? Explain how you know.
For the function which is linear, find the formula that fits the data.

h(x) is linear because $\frac{\Delta h}{\Delta x} = \frac{-4}{1}$ = constant

$y = mx + b$

$y = -4x + 20 \Rightarrow \boxed{h(x) = -4x + 20}$

(b) Which (if any) of these functions is an exponential function? Explain how you know.
For the function(s) which is exponential, find the formula that fits the data.

g(x) is exponential ⇒ Ratio of y values is approx constant

$\boxed{g(x) = 22.49(2.1)^x}$

$\frac{10.71}{5.1} = 2.1$ $\frac{22.49}{10.71} \simeq 2.1$

$\frac{47.23}{22.49} \simeq 2.1$ $\frac{99.18}{47.23} \simeq 2.1$

9. (5 pts) Determine whether the given function is continuous on the interval.
If the function is not continuous on the interval, give the point of discontinuity.

(a) Is $f(x) = \frac{x^3}{x-3}$ continuous on [0,2]? yes

(b) Is $g(t) = \frac{e^{-2t}}{e^{-2t}+3}$ continuous on **R** (all real numbers)? yes

(c) Is $h(t) = \tan(2x)$ continuous on $\left[0, \frac{\pi}{4}\right]$? no, pt of discontinuity at $x = \frac{\pi}{4}$

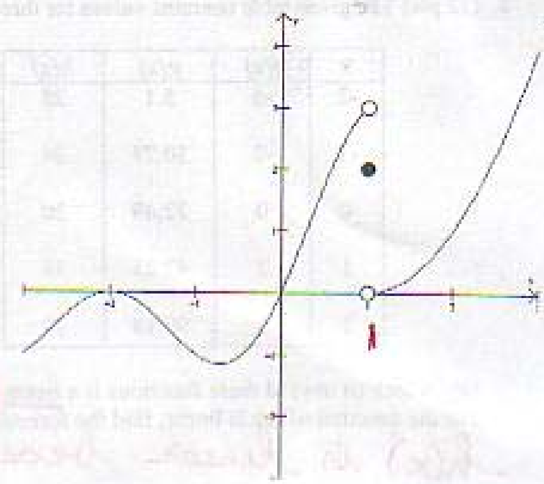
10. (6 pts) What are the 3 criteria, based on the FORMAL definition of continuity, that a function, *f*, must meet in order to be continuous at a point x = c ?

I. f(c) must exist

II. $\lim_{x \to c} f(x)$ must exist

III. $\lim_{x \to c} f(x) = f(c)$

11. (8 pts) (Use the given graph of $y = f(x)$ to find the following limits, if they exist. Put "dne" for limits that don't exist.



(a) $\lim_{x\to 1^+} f(x) =$ 0

(b) $\lim_{x\to 1^-} f(x) =$ 3

(c) $\lim_{x\to 1} f(x) =$ DNE

(d) $\lim_{x\to -2} f(x) =$ 0

12. (4 pts) Referring to the function shown in the graph for problem #11, explain, in terms of the FORMAL definition of continuity, why the function is not continuous at the point x = 1.

I) f(1) does exist (f(1) = 2)

II) BUT $\lim_{x\to 1} f(x)$ DNE

$\therefore$ f is not continuous at x = 1

13. (4 pts) Use the given table to evaluate the limit, if it exists. Put "dne" if the limit does not exist.

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x) = \frac{\sin(2x)}{x}$	1.986693	1.999867	1.999999	undefined	1.999999	1.999867	1.986693

$\lim_{x\to 0} \frac{\sin(2x)}{x} =$ 2

14. (10 pts) Find the following limits analytically (using algebraic techniques):

(a) $\lim_{x\to\infty} \frac{(3x+1)\frac{1}{x^2}}{(9x^2+3x-1)\frac{1}{x^2}} =$ 0

$$\lim_{x\to\infty} \frac{\frac{3}{x}+\frac{1}{x^2}}{9+\frac{3}{x^2}-\frac{1}{x^2}} = \frac{0+0}{3+0+0} = 0$$

(b) $\lim_{x\to\infty} \frac{(7x+4)\frac{1}{x}}{(9x-5)\frac{1}{x}} =$ 7/9

$$\lim_{x\to\infty} \frac{7+\frac{4}{x}}{9-\frac{5}{x}} = \frac{7+0}{9-0} = \frac{7}{9}$$

(c) $\lim_{x\to 2} \frac{x^2-4}{x-2} =$ 4

$$\lim_{x\to 2} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}} = \lim_{x\to 2} (x+2) = 4$$

(d) $\lim_{x\to 1} \frac{x-1}{x^2-3x+2} =$ −1

$$\lim_{x\to 1} \frac{\cancel{(x-1)}}{\cancel{(x-1)}(x-2)} = \lim_{x\to 1} \frac{1}{x-2} = \frac{1}{1-2} = -1$$

(e) $\lim_{x\to -1} 5x^2+3x+2 =$ 4

$$\lim_{x\to -1} 5x^2+3x+2 = 5(-1)^2+3(-1)+2$$
$$= 5-3+2 = 4$$