Math 265A: Test 5 **Name:** KEY

Please organize your work in a clear, logical manner. Credit is based on the amount of correct work shown, not on the final answer. You will need a graphing calculator for this exam. Calculators with a Computer Algebra System are NOT allowed. *Simplify your answers as much as possible.*

1. Find the general antiderivatives:

a) $\int z - 3\cos(z)\, dz$

$= \boxed{\frac{1}{2}z^2 - 3\sin(z) + C}$

b) $\int 1 + \sin(t)\, dt$

$= \boxed{t - \cos(t) + C}$

c) $\int \frac{7}{x} - \frac{1}{x^7} + \sqrt[7]{x}\, dx = \int \frac{7}{x} - x^{-7} + x^{1/7}\, dx$

$= \boxed{7\ln|x| + \frac{1}{6}x^{-6} + \frac{7}{8}x^{8/7} + C}$

2. Use calculus (not your calculator) to evaluate the given definite integrals. Give your answer in exact terms.

(a) $\int_1^3 -6t^2 + 4t - 5\, dt = -2t^3 + 2t^2 - 5t \Big|_1^3$

$= -2(3)^3 + 2(3)^2 - 5(3) - [-2(1)^3 + 2(1)^2 - 5(1)]$

$= \boxed{46}$

(b) $\int_0^w b_1 + \frac{b_2 - b_1}{w} x\, dx = b_1 x + \frac{b_2 - b_1}{w} \frac{1}{2} x^2 \Big|_{x=0}^{x=w}$

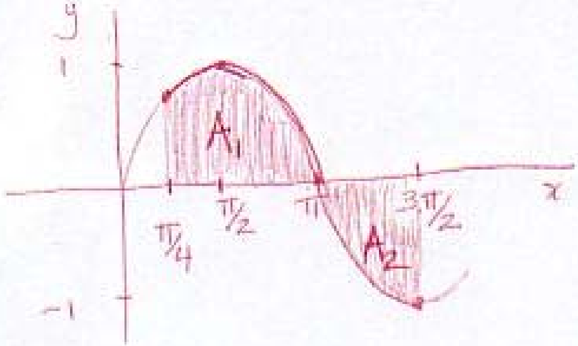
$= b_1 w + \frac{b_2 - b_1}{w} \cdot \frac{1}{2} w^2$

$= b_1 w + \frac{1}{2} b_2 w - \frac{1}{2} b_1 w = \boxed{\frac{1}{2} b_1 w + \frac{1}{2} b_2 w}$

3. Use calculus (not your calculator) to find the exact area between $f(x) = \sin x$ and the x-axis on the interval $\left[\frac{\pi}{4}, \frac{3\pi}{2}\right]$

Sketch a graph that illustrates the area you're finding.

Leave your answer in EXACT terms (there should be a square root in your answer).



Area = Area 1 + Area 2

$$= \int_{\pi/4}^{\pi} \sin(x)\,dx + -\int_{\pi}^{3\pi/2} \sin(x)\,dx$$

$$= -\cos(x)\Big|_{\pi/4}^{\pi} + \cos(x)\Big|_{\pi}^{3\pi/2}$$

$$= -\cos(\pi) - \left(-\cos\frac{\pi}{4}\right) + \cos\left(\frac{3\pi}{2}\right) - \cos(\pi)$$

$$= -(-1) + \frac{1}{\sqrt{2}} + 0 - (-1)$$

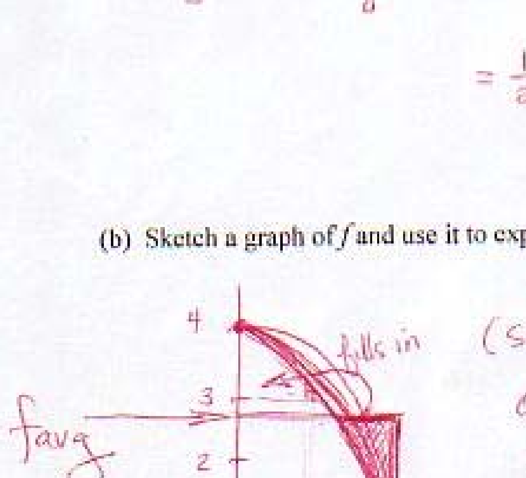
$$= \boxed{2 + \frac{1}{\sqrt{2}} \text{ square units}}$$

4. (a) Use calculus to find the average value of $f(x) = 4 - x^2$ on the interval from $[0, 2]$.

$$f_{avg} = \frac{1}{2-0}\int_0^2 4 - x^2\,dx = \frac{1}{2}\left[4x - \frac{1}{3}x^3\right|_0^2$$

$$= \frac{1}{2}\left[8 - \frac{8}{3}\right] = \frac{8}{3} = 2\frac{2}{3}$$

(b) Sketch a graph of f and use it to explain why the value you found in part (a) is reasonable.



You can either use an area argument (showing $f_{avg} \cdot (b-a) = \int_a^b f(x)\,dx$)

OR argue that there are "more" y-values in the upper region of the graph than in the lower region.

5. A car going 30 ft/sec brakes to a stop in 2 seconds. Its velocity is recorded every half second and is given in the following table:

t (seconds)	0	.5	1	1.5	2
v(t) (ft/sec)	30	18	8	2	0

Using a Left Sum and Right Sum, give an upper and lower estimate of the total distance traveled by the car during the 2 seconds.

Left Sum: Distance $\cong (30+18+8+2)\cdot(.5)$

$= 58\cdot\left(\frac{1}{2}\right) = 29$ feet

Right Sum: Distance $\cong (18+8+2+0)(.5)$

$= 28\left(\frac{1}{2}\right) = 19$

Upper estimate: 29 feet

Lower estimate: 19 feet

6. The definition of the definite integral includes the following equation:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x$$

a) If $f(x) \geq 0$ for all x, sketch a graph that illustrates $\int_a^b f(x)dx$

Area between y=f(x) and the x-axis is given by $\int_a^b f(x)dx$

b) If n = 4, fill in, on your graph from part (a), the geometric figures whose total area is given by $\sum_{k=1}^{n} f(x_k)\Delta x$

c) What does $f(x_k)$ represent in terms of these geometric figures? The height of each rectangle

d) What happens to Δx in the limit? It shrinks to zero (infinitesimally small.)

7. Find the following:

(a) $\frac{d}{dx}\int_0^x t^4 - 3t\ dt = \boxed{x^4 - 3x}$

(b) $\frac{d}{dx}\int_0^x e^{-t^2}\ dt = \boxed{e^{-x^2}}$

8. The rate (cm per day) at which the diameter of a grapefruit changes over time as the grapefruit grows is given by the function $r(t) = 0.4e^{-0.001t^2}$, where t is the number of days from pollination.

(a) Use your calculator to find the value of $\int_0^{21} r(t)dt$

$\int_0^{21} r(t)dt \approx$ 7.3128

(b) Explain in practical terms what the quantity $\int_0^{21} r(t)dt$ represents. Include units!

units are $\frac{cm}{day} \cdot days = cm$

During the 21 day period since pollination, the diameter of the grapefruit has increased by ≈ 7.3 cm

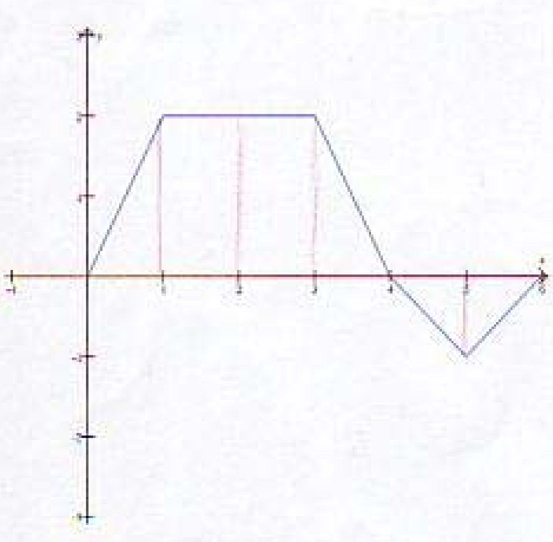
9. Use the given graph of F' to do the following:

(a) Fill in the following data table:

x	0	1	2	3	4	5	6
$F(x)$	1	2	4	6	7	6.5	6

(b) Construct the graph of F on the interval from $0 \le x \le 6$. Label any local max's or min's and point(s) of inflection.

Graph of F'



Graph of F