Math 265B Spring 2020 **Name:______________________________**

100 points

INSTRUCTIONS: For full or partial credit, all solutions must be neatly written with all of the details clearly presented. Answers given without adequate justification may not receive credit. Make sure you are using correct notation. Box answers where appropriate.

1. (2 pts) What is the difference between a <u>sequence</u> and a <u>series</u>?

2. (4 pts) (a) What does it mean for a sequence to converge?

(b) What does it mean for a series to converge?

3. (10 pts) Consider the series $\sum_{k=1}^{\infty} \frac{1}{2k+1} - \frac{1}{2k+3}$

(a) Find the first 3 partial sums for the series

(b) What is the name of this type of series? ____________________________________

(c) Find a formula for the nth partial sum of the series

(d) Does the series converge or diverge? If it converges, what value does it converge to? (Your work should include a limit!)

4. (5 pts) True or false (circle the correct answer):

(a) If $0 \le a_n \le b_n$ and $\sum_{n=1}^{\infty} a_n$ converges then $\sum_{n=1}^{\infty} b_n$ converges also. True False

(b) If $a_n \le b_n \le 0$ and $\sum_{n=1}^{\infty} b_n$ diverges then $\sum_{n=1}^{\infty} a_n$ diverges also. True False

(c) For a series $\sum_{n=1}^{\infty} a_n$ if you find that $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$, then you can conclude the series diverges. True False

(d) The interval of convergence of a power series gives the x-values for which the series diverges: True False

(e) The formula for Taylor polynomials was created by making a function and its derivatives and the polynomial and its derivatives match at a particular x-value. True False

5. (15 pts) Fill in and/ or circle the correct answer:

(a) $\sum_{k=1}^{\infty} \frac{1}{k^{3/2}}$ converges / diverges (circle one) because ____

(b) $\sum_{k=0}^{\infty} \frac{1}{3}\left(\frac{5}{4}\right)^k$ converges / diverges (circle one) because ____

(c) If $\sum_{k=1}^{\infty} a_k$ converges conditionally, then $\sum_{k=1}^{\infty} |a_k|$ must converge / diverge (circle one)

(d) The series $\sum_{k=1}^{\infty} \frac{1}{k}$ is called the _________________________ series and it converges / diverges (circle one).

(e) Does the series $\sum_{k=1}^{\infty} \frac{(-1)^k}{k}$ converge absolutely? ____________ Show work or explain how you know (no proof necessary!).

Does $\sum_{k=1}^{\infty} \frac{(-1)^k}{k}$ converge conditionally?___________ Show work to justify your answer.

6. (6 pts) Find the sum of the series $\sum_{k=0}^{\infty} \frac{2^k + 7}{3^k}$

7. (6 pts) Use the Integral Test to <u>prove</u> that the series either converges or diverges. Be sure to check the conditions (show work!) and clearly state your conclusion.

$$\sum_{k=1}^{\infty} \frac{2k}{e^{k^2}}$$

8. (6 pts) Use the Limit Comparison Test to prove that the series $\sum_{k=1}^{\infty} \frac{3k^2 + k}{k^5 + 5k^2 + 2}$ converges or diverges.

Clearly state your conclusion!

9. (4 pts) Use any test to prove the series $\sum_{k=1}^{\infty} \frac{5k^3}{k^3 + 10k}$ converges or diverges. Clearly state your conclusion.

10. (8 pts) (a) Write the following series using $\sum$ form: $\frac{1}{2}-\frac{3}{4}+\frac{5}{8}-\frac{7}{16}+\cdots$

(b) Determine whether the series converges or diverges, then <u>prove</u> your guess using an appropriate test.

11. (12 pts) Given the following power series, determine the radius and interval of convergence (you do not have to test the endpoints of the interval).

(a) $\sum_{n=1}^{\infty} \frac{x^n}{(2n)!}$

R = ___________ Interval = __________________________

(b). $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n}$

R = ___________ Interval = __________________________

12. (6 pts) Use the Taylor Polynomial (derivative) formula to find the Taylor polynomial of degree 3 about x = 2 for the function $\ln(x)$

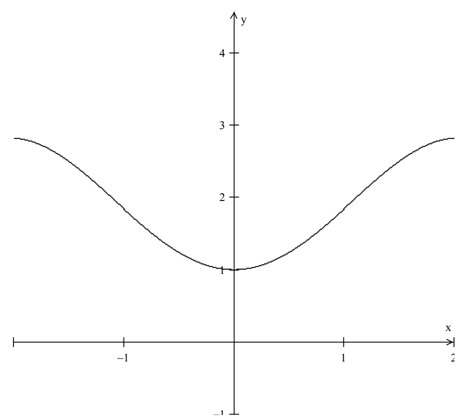
13. (3 pts) Suppose $P_2(x) = a + bx + cx^2$ is the second degree Taylor polynomial for the function *f* about x = 0. What can you say about value of a, b and the sign of c, based the graph of *f*?

Value of a:________________________

Value of b:________________________

Sign of c:________________________

Graph of *f*



14. (3 pts) Write down (you do not have to find them from the derivative formula!) the Taylor series (aka MacLaurin series), expanded about x = 0, for the following functions. Write them in expanded form <u>and</u> using $\sum$ notation.

a. e^x

b. sin(x)

c. cos(x)

15. (10 pts) (a) Use the Taylor series for e^x to determine the Taylor Series for $e^{i\theta}$. Find the first 7 terms of the series and simplify the result. You don't need to write the general term or use sigma notation.

(b) Partition the series above into real terms and imaginary* terms (*the terms with an i in them after simplifying)

(c) Identify the series for the real part and for the imaginary part (factor out the i in the imaginary terms first).

(d) Based on the work above, write an equation that relates $e^{i\theta}$, $\sin\theta$, and $\cos\theta$

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(This relationship is called "Euler's Equation".