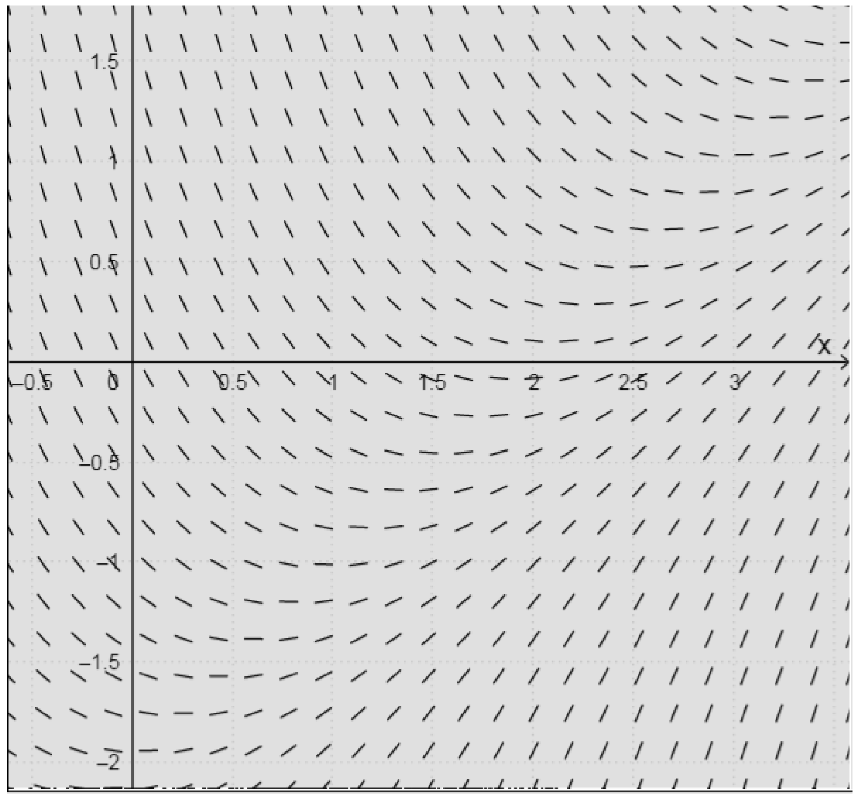
Math 265B: Euler's Method

Goal: Estimate the solution to the IVP $y'(x) = x - y - 2, \quad y(0) = 1$ at y(2)

Make a "Polygonal Approximation" of the solution curve



Euler's Method:

We want to estimate $y\left(x_{final}\right)$ given the IVP $y' = f(x,y), \quad y(x_0) = y_0$

Process:

1. Locate a **point** (start at the IC, (x_0, y_0)).
2. Find the **slope** of the solution curve at that point, using $y' = \frac{dy}{dx} = slope$
3. Find Δy: $\Delta y = m \cdot \Delta x$
4. **Project** forward to a new point, using the fact that $x_{k+1} = x_k + \Delta x$ and $y_{k+1} = y_k + \Delta y$
5. Continue this process until you reach the target x-value.

- Each iteration of this process is called a "**step**".
- The Δx value is called the "step-size". (The variable *h* is more commonly used for step-size.)
- The number of steps needed is given by $\frac{x_{final} - x_{initial}}{\Delta x}$

Step	**Point**		m = **Slope**	Change in y	**Project** to New Point	
k	x_k	y_k	$y'(x) = f(x_k, y_k)$	$\Delta y = m \cdot \Delta x$	$x_{k+1} = x_k + \Delta x$	$y_{k+1} = y_k + \Delta y$

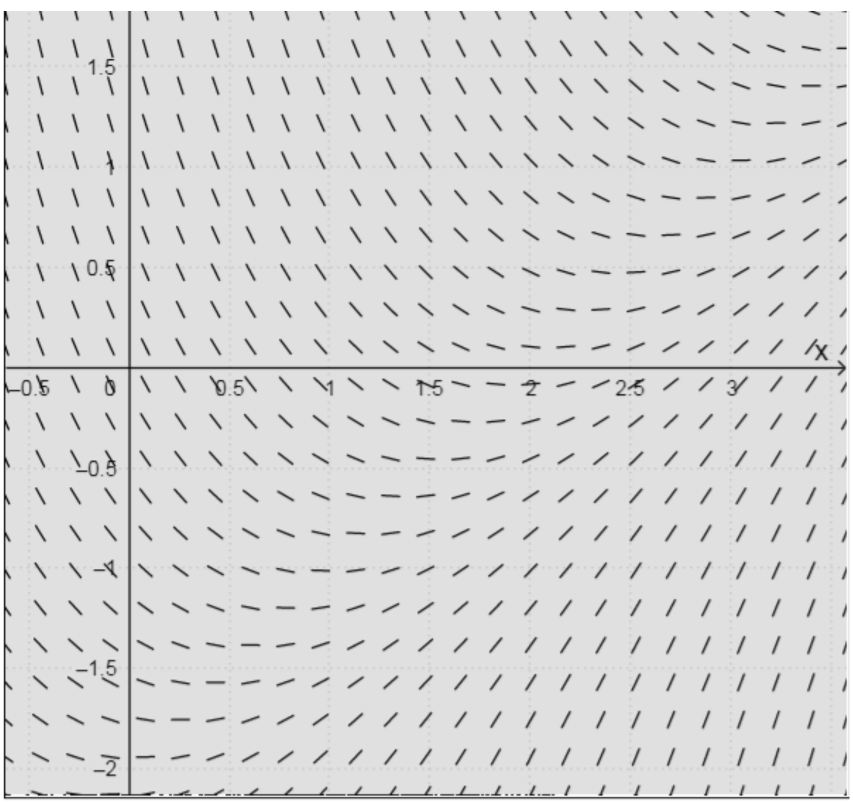
Example: Use Euler's method, by hand, to estimate the solution to the IVP $y' = x - y - 2,\ y(0) = 1$ at $y(2)$ using the following step sizes: , and $\Delta x = .5$ Illustrate the polygonal approximation on the slope field.

$\Delta x = 1$

Step	Point		Slope	Change in y	Project to New Point	
k	x_k	y_k	$y' = x - y - 2$	$\Delta y = m \cdot \Delta x$	$x_k + \Delta x$	$y_k + \Delta y$

$\Delta x = .5$

Step	Point		Slope	Change in y	Project to New Point	
k	x_k	y_k	$y'(x)$	$\Delta y = m \cdot \Delta x$	$x_k + \Delta x$	$y_k + \Delta y$



Now use the GeoGebra program to confirm your answers above and to continue the process $\Delta x = .1$ $\Delta x = .05$

Error Analysis

Put all of your estimations in the table below.

Then determine the actual value of y(2) using the fact that the analytical solution to the IVP is $y = x + 4e^{-x} - 3$.*

**Note: Wolfram will solve the IVP for you. Use the command "Solve" followed by the IVP*

Finally, determine the error. Remember: Error = Estimate – Actual

Δx	Euler's Estimate y(2)	Actual value of y(2)	Error

If the step size is reduced by a factor of 10, what effect does this appear to have on the error?

How efficient is this process?