This last example illustrated a type of DE that is called "autonomous". These DE's are of the form $y' = f(y)$, meaning there's no "t" in the slope function formula.

Autonomous DE's are really useful for modeling real problems, which we'll see later.

Why are they useful? Well, first off, they have **equilibrium solutions**! These solutions describe long-term (in the long run) asymptotic behavior which is common in any situation where something is changing dynamically at first then settles into some kind of steady state over time.

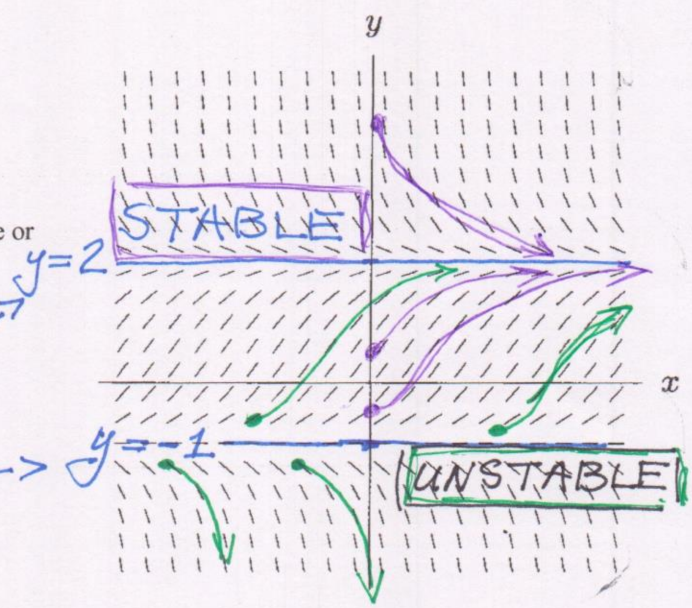
Equilibrium Solution: A solution that is constant for all values of t. The graph is a horizontal line. Equilibrium solutions can be determined by setting $y' = 0$ to zero and solving for y.

Stable Equilibrium Solution: Solution curves are pulled toward the horizontal line as t goes to infinity.

Unstable Equilibrium Solution: Solution curves are move away from the horizontal line as t goes to infinity.

Example: Consider the following slope field for the differential equation $y' = 0.5(1+y)(2-y)$.

(a) Is it an Autonomous DE? How can you tell?
(b) What are the equilibrium solutions? Find them algebraically and graphically.
(c) Determine whether each equilibrium solution is stable or unstable.
(d) Solve the DE using Separation of Variables (find the General Solution)



(a) Yes this DE is autonomous because there is no "t" in the slope function.

(b) Equil Solutions:

$y' = 0$

$\Rightarrow .5(1+y)(2-y) = 0$

$\boxed{y=-1}$ $\boxed{y=2}$

(c) By inspection, we can see $y=2$ is stable because solution curves are drawn toward $y=2$ as $t \to \infty$

We can see $y=-1$ is unstable because solution curves are ~~drawn~~ pushed away from $y=-1$

(d) Solve $y' = 0.5(1+y)(2-y)$

Partial Fraction Decomposition

Tip: leave constant factor with dt

$$\frac{dy}{dt} = .5(1+y)(2-y)$$

$$\frac{dy}{(1+y)(2-y)} = .5\,dt$$

Tip: factor out neg in 2-y factor

$$\frac{dy}{-(y-2)(y+1)} = .5\,dt$$

$$\int \frac{dy}{(y-2)(y+1)} = \int -.5\,dt$$

$$\int \frac{1}{(y-2)(y+1)}\,dt$$

$$= \int \frac{A}{y-2} + \frac{B}{y+1}\,dt$$

$$= \int \frac{1/3}{y-2} + \frac{-1/3}{y+1}\,dt$$

$$= \frac{1}{3}\ln|y-2| - \frac{1}{3}\ln|y+1|$$

$$\frac{1}{(y-2)(y+1)} = \frac{A}{y-2} + \frac{B}{y+1}$$

$$1 = A(y+1) + B(y-2)$$

$$y = 2 \Rightarrow 1 = 3A$$

$$A = \frac{1}{3}$$

$$y = -1 \Rightarrow 1 = -3B$$

$$B = -\frac{1}{3}$$

$$\frac{1}{3}\ln|y-2| - \frac{1}{3}\ln|y+1| = -.5t + C$$

$$\frac{1}{3}\ln\left|\frac{y-2}{y+1}\right| = -.5t + C$$

$$\ln\left|\frac{y-2}{y+1}\right| = -1.5t + C$$

$$e^{\ln\left|\frac{y-2}{y+1}\right|} = e^{-1.5t + C}$$

$$\left|\frac{y-2}{y+1}\right| = e^{C} \cdot e^{-1.5t}$$

$$\frac{y-2}{y+1} = \pm e^{C} e^{-1.5t}$$

$$\frac{y-2}{y+1} = C_1 e^{-1.5t}$$

Solve for y. Tricky, kind of messy algebra

$$\boxed{y = \frac{2 + C_1 e^{-1.5t}}{1 - C_1 e^{-1.5t}}}$$