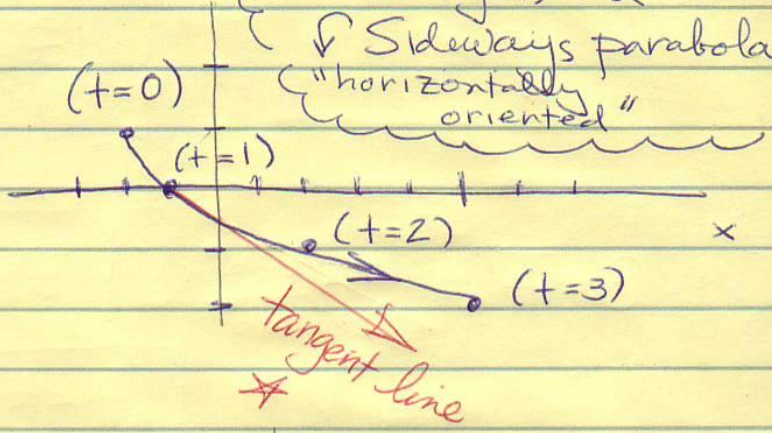
Chapter 11 Quiz Key

1. $x = t^2 - 2 \quad 0 \le t \le 3$.
$y = 1 - t$

(c) $t = 1 - y$
$\Rightarrow x = (1-y)^2 - 2$
or $x = (y-1)^2 - 2$
↑ Sideways parabola ("horizontally oriented")

(a)

t	x	y
0	-2	1
1	-1	0
2	2	-1
3	5	-2



(b) Tangent Line Equation (parametric form) at $t = a$

$x = x_0 + (\Delta x)(t-a)$

$y = y_0 + (\Delta y)(t-a)$

where (x_0, y_0) = point of tangency

$\Delta x = \left.\frac{dx}{dt}\right|_{t=a}$

$\Delta y = \left.\frac{dy}{dt}\right|_{t=a}$

Point of tangency is $(-1, 0)$

$\left.\frac{dx}{dt}\right|_{t=1} = 2t\Big|_{t=1} = 2$

$\left.\frac{dy}{dt}\right|_{t=1} = -1\Big|_{t=1} = -1$

So ...

$x = -1 + 2(t-1)$
$y = 0 - 1(t-1)$
↑ "phase shift"

★
$$\begin{cases} x = -1 + 2(t-1) \\ y = -(t-1) \end{cases} \quad t \ge 1$$

2 (a) $P(3,5)$ to $Q(-1,2)$

$(x_0, y_0) = (3,5)$
$\Delta x = -4$
$\Delta y = -3$

$\Rightarrow$ Line Segment Param:

$$\begin{cases} x = 3 - 4t \\ y = 5 - 3t \\ 0 \le t \le 1 \end{cases}$$

(b) (No work necessary but following is the line of reasoning:

1) Circle (center $(0,0)$)
Radius $= 2$
ccw orientation

$$\begin{cases} x = 2\cos t \\ y = 2\sin t \\ 0 \le t \le 2\pi \end{cases}$$

2) Adjust period
$0 \le t \le 2\pi$
to $0 \le t \le 10$

$$\begin{cases} x = 2\cos\left(\frac{\pi}{5}t\right) \\ y = 2\sin\left(\frac{\pi}{5}t\right) \end{cases}$$

3) Shift center to $(5,1)$

$$\begin{cases} x - 5 = 2\cos\left(\frac{\pi}{5}t\right) \\ y - 1 = 2\sin\left(\frac{\pi}{5}t\right) \\ 0 \le t \le 10 \end{cases}$$

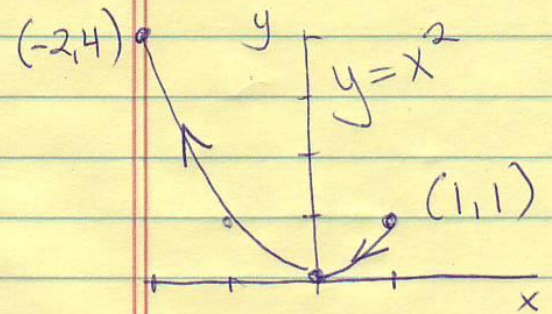
4) Reverse orientation

$$x - 5 = 2\cos\left(\frac{\pi}{5}t\right)$$
$$y - 1 = -2\sin\left(\frac{\pi}{5}t\right)$$
$$0 \le t \le 5$$

Final Param:

$$\begin{cases} x = 5 + 2\cos\left(\frac{\pi}{5}t\right) \\ y = 1 - 2\sin\left(\frac{\pi}{5}t\right) \\ 0 \le t \le 10 \end{cases}$$

2 (c) TYPO: (1,0) should be (1,1)



First attempt: Let $x=t$

then $y=t^2$

$-2 \le t \le 1$

This param. describes the correct piece of the parabola but the orientation is wrong

Let $-t$ be substituted for t to change direction:

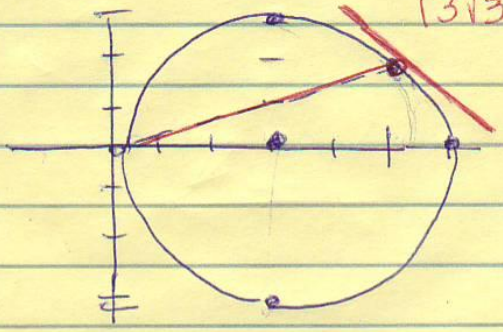
$$\begin{cases} x = -t \\ y = (-t)^2 \\ -2 \le -t \le 1 \end{cases}$$

$$\Rightarrow \boxed{\begin{cases} x = -t \\ y = t^2 \\ -1 \le t \le 2 \end{cases}}$$

This gives the correct orientation

3 (a) $r = 6\cos(\theta)$

$r = 2a\cos\theta \Rightarrow$ circle: center $(3,0)$ Radius $= 3$



$m = \frac{dy}{dx} = -\sqrt{3} \approx -1.7$

$r = 6\cos\theta$

$r' = -6\sin\theta$

(b) $\frac{dy}{dx} = \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta}$

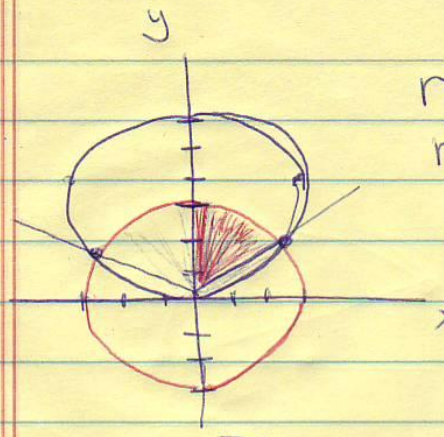
$= \frac{-6\sin\theta\sin\theta + 6\cos\theta\cos\theta}{-6\sin\theta\cos\theta - 6\cos\theta\sin\theta}$

$= \frac{\cos^2\theta - \sin^2\theta}{-2\sin\theta\cos\theta} = -\frac{\cos 2\theta}{\sin 2\theta}$

$$\boxed{\frac{dy}{dx} = -\tan 2\theta}$$

so $\left.\frac{dy}{dx}\right|_{\theta=\pi/6} = -\tan 2\left(\frac{\pi}{6}\right) = -\tan\left(\frac{\pi}{3}\right) = -\sqrt{3}$

4.



Point of intersection

$$\left.\begin{aligned} r &= 6\sin\theta \\ r &= 3 \end{aligned}\right\} \Rightarrow 3 = 6\sin\theta$$

$$\frac{1}{2} = \sin\theta$$

$$\theta = \pi/6$$

$$A_{\frac{1}{2}} = \int_0^{\pi/6} \frac{1}{2}\left[6\sin\theta\right]^2 d\theta + \int_{\pi/6}^{\pi/2} \frac{1}{2}\left[3\right]^2 d\theta$$

$$= 18\int_0^{\pi/6} \sin^2\theta\, d\theta + \int_{\pi/6}^{\pi/2} \frac{9}{2}\, d\theta$$

$$= 9\int_0^{\pi/6} 1-\cos 2\theta\, d\theta + 9/2\,\theta \Big|_{\pi/6}^{\pi/2}$$

$$= 9\theta - \frac{9}{2}\sin 2\theta \Big|_0^{\pi/6} + \frac{9}{2}\left[\frac{\pi}{2} - \frac{\pi}{6}\right]$$

$$= 9\left(\frac{\pi}{6}\right) - \frac{9}{2}\sin\left(\pi/3\right) - \cancel{9(0)} + \cancel{\frac{9}{2}\sin 0} + \frac{9}{2}\left[\frac{2\pi}{6}\right]$$

$$= \frac{9\pi}{3} - \frac{9}{2}\frac{\sqrt{3}}{2} = \boxed{3\pi - \frac{9\sqrt{3}}{4}} \quad \tfrac{1}{2} \text{ area}$$

So $\boxed{A = 2A_{\frac{1}{2}} = 6\pi - \frac{9\sqrt{3}}{2}}$

Total area