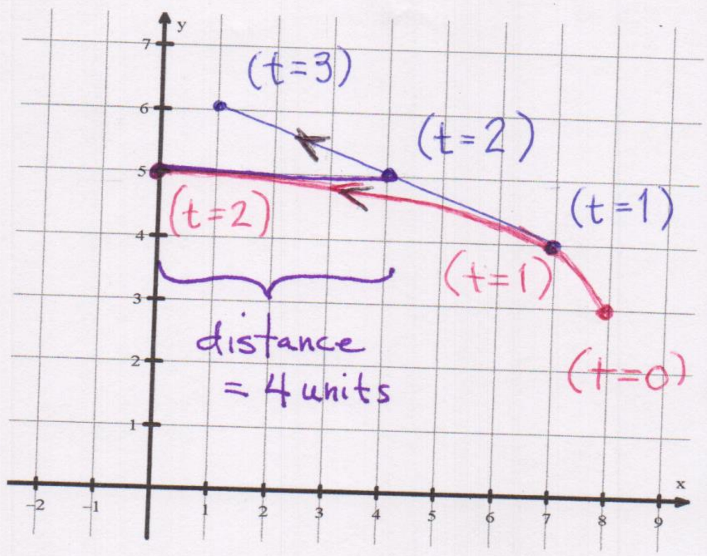
Math 265B: Chapter 11 Quiz (25 points) Name: KEY

Show work for credit!

1. (8 points) Consider the curve described by the parameterization $\begin{cases} x = 8 - t^3 \\ y = t + 3 \end{cases}$ $0 \le t \le 2$, t = time in seconds

(a) Graph the curve by making a t,x,y chart. Show the **orientation** of the curve (use an arrow) and **label** the t-values for each point in your graph.

t	x	y
0	8	3
1	(7,	4)
2	0	5



(b) Find the slope of the tangent line at the point where t = 1.

$$\left.\frac{dy}{dx}\right|_{t=1} = \left.\frac{dy/dt}{dx/dt}\right|_{t=1} = \left.\frac{1}{-3t^2}\right|_{t=1} = -\frac{1}{3}$$

$$\boxed{m_{tan} = -\frac{1}{3}}$$

$$\left.\frac{dx}{dt}\right|_{t=1} = -3$$

$$\frac{dy}{dt} = 1$$

(c) Find the parametric form of the tangent line at the point where t = 1.

$$x = x_0 + \left.\frac{dx}{dt}\right|_{t=1}(t-1)$$

$$y = y_0 + \left.\frac{dy}{dt}\right|_{t=1}(t-1)$$

$$x_0 = x(1) = 7 \qquad y_0 = y(1) = 4$$

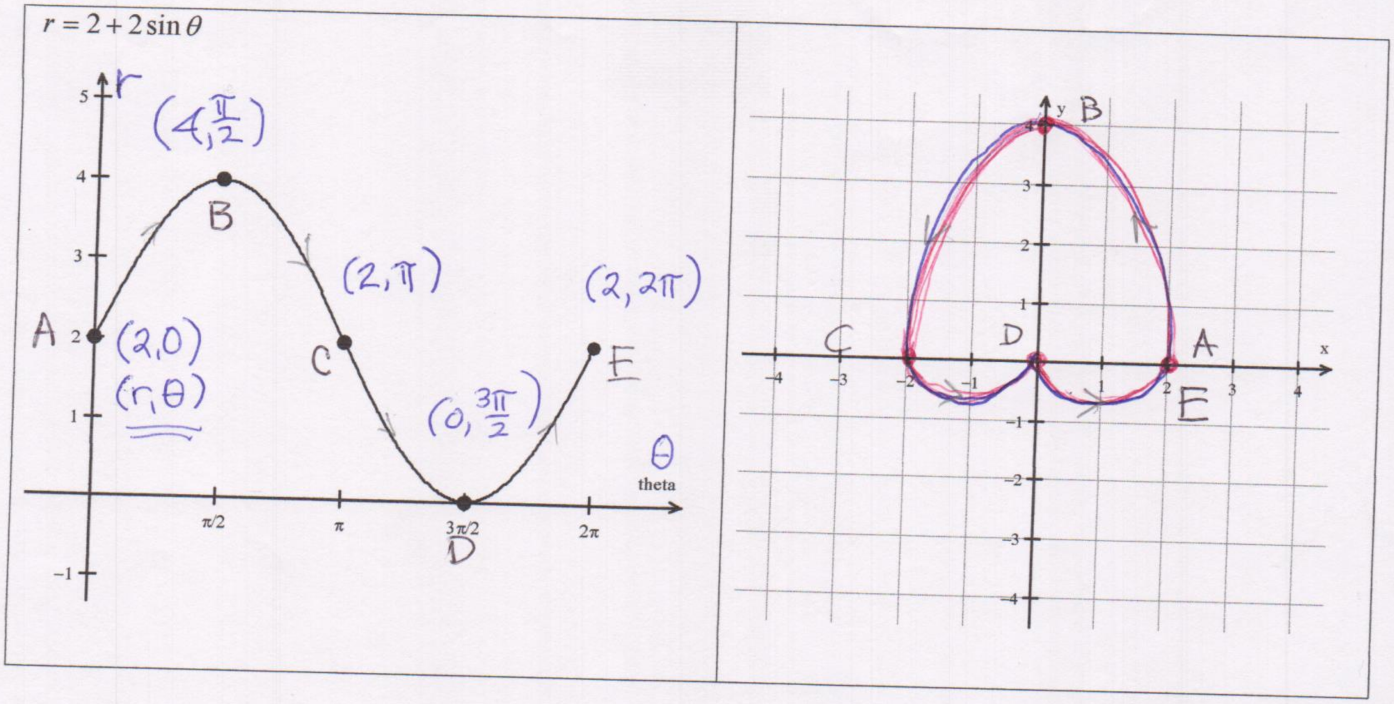
$$\boxed{\begin{aligned} x &= 7 - 3(t-1) \\ y &= 4 + 1(t-1) \end{aligned}}$$

(d) Sketch an accurate graph of the tangent line on your graph from part (a). Include at least 2 points on the tangent line.

(e) If the graph of the curve and tangent line describes a particle that split into two pieces at t = 1, with one piece following the original path and the other following the tangential path, determine how far apart the pieces are at t = 2 seconds. (1 second after the split occurred). Show this distance on your graph.

distance apart = 4 units (by inspection)

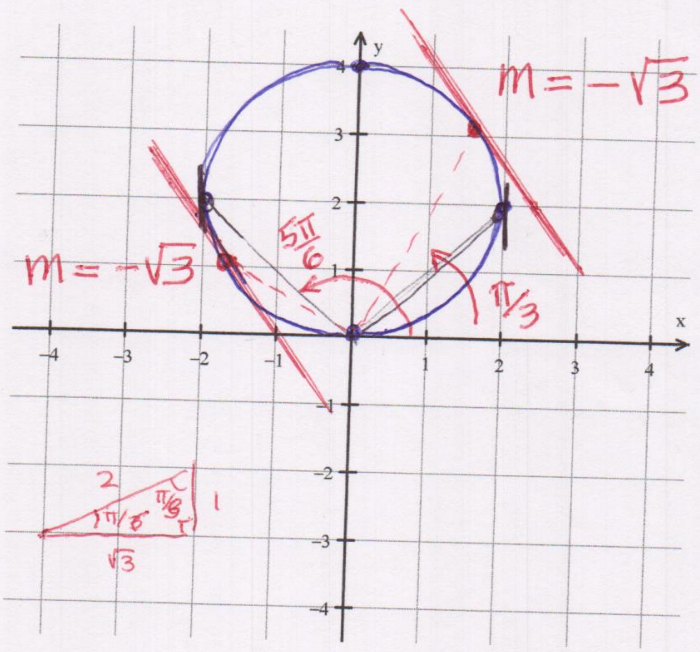
2. (3 points) Use the Cartesian r,θ graph to graph the polar equation in the xy-coordinate system. Label the values of the points shown in the r,θ graph and clearly show them on the x,y graph.



3. (8 pts) Given the polar equation $r = 4\sin\theta$

(a) Sketch the graph of the equation in the xy-coordinate system.

(b) Find the slope of the line tangent to the graph at $\theta = \frac{\pi}{3}, \frac{5\pi}{6}$. Sketch tangents at these points.



$$\frac{dy}{dx} = \frac{\frac{d}{d\theta}(r\sin\theta)}{\frac{d}{d\theta}(r\cos\theta)} \quad \frac{dy/d\theta}{dx/d\theta}$$

$$= \frac{\frac{d}{d\theta}(4\sin^2\theta)}{\frac{d}{d\theta}(4\sin\theta\cos\theta)}$$

$$= \frac{8\sin\theta\cos\theta}{4\cos\theta\cos\theta - 4\sin\theta\sin\theta}$$

$$= \frac{4\sin 2\theta}{4\cos 2\theta} = \tan 2\theta$$

$$\left.\frac{dy}{dx}\right|_{\theta=\pi/3} = \left.\tan 2\theta\right|_{\theta=\frac{\pi}{3}} = \tan\left(\frac{2\pi}{3}\right) = -\frac{\sqrt{3}}{1}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{5\pi}{6}} = \left.\tan 2\theta\right|_{\theta=\frac{5\pi}{6}} = \tan\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{1}$$

3 (continued)

(c) Algebraically determine the values of θ where the tangent line is vertical. Show work!

$$2\theta = \frac{1\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

$\frac{dy}{dx}$ is undefined when $\cos(2\theta) = 0$

$$2\theta = \frac{(2n-1)\pi}{2}, \; n \in \mathbb{N}$$

$n = 1, 2, 3, \ldots$

$$2\theta = \frac{(2n-1)\pi}{2}$$

1, 3, 5, 7

$$\theta = \frac{(2n-1)\pi}{4}, \; n \in \mathbb{Z}$$

$$\boxed{\text{specifically } \theta = \frac{\pi}{4}, \frac{3\pi}{4}}$$

4. (6 points) Find the exact area of the region that is inside the curve $r = \sqrt{2}$ and also inside the curve $r = 2\cos\theta$. Use calculus to find the area.

$$A = 2\left[\int_0^{\pi/4} \frac{1}{2}(\sqrt{2})^2 d\theta + \int_{\pi/4}^{\pi/2} \frac{1}{2}[2\cos\theta]^2 d\theta\right]$$

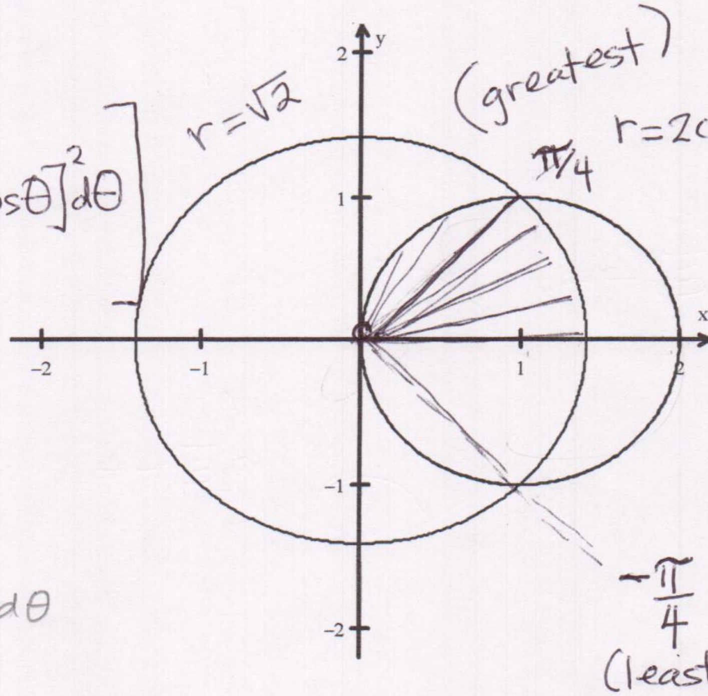
$$= \int_0^{\pi/4} 2\,d\theta + \int_{\pi/4}^{\pi/2} 4\cos^2\theta\, d\theta$$

$$= 2\theta\Big|_0^{\pi/4} + \left[4\int_{\pi/4}^{\pi/2} \frac{1}{2} + \frac{1}{2}\cos 2\theta\, d\theta\right.$$

$$= \frac{\pi}{2} - 2\theta + \sin 2\theta \Big|_{\pi/4}^{\pi/2}$$

$$= \frac{\pi}{2} + \left(\pi - \frac{\pi}{2}\right) + \left.\left(\overset{0}{\cancel{\sin\pi}} - \sin\frac{\pi}{2}\right)\right]$$

$$= \pi - 1$$



Points of intersection

$2\cos\theta = \sqrt{2}$ | algebraic

$\cos\theta = \frac{\sqrt{2}}{2}$ | By inspection

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$

not useful

instead $\theta = -\frac{\pi}{4}$