Flowchart for choosing a Convergence Test for an infinite series, $\sum_{k=1}^{\infty} a_k$

Step 1: Look at the terms in the series; i.e., expand a few or just look at the form of a_k .

Vital question: **If the terms are all positive**, will these terms get small FAST as k gets large?
What does "dominance" say about this? Is the numerator or denominator growing faster?

Note: If the terms are alternating, then you'll be looking at the Alternating Series Test, then judging absolute vs. conditional convergence.

Step 2: Make an educated guess about whether the series converges or diverges.

Step 3: PROVE your guess by applying the appropriate test.
Be sure to check all conditions BEFORE applying the test.

Start Here:		**Test to Apply**	**Result**	**Conclusion**
Do the terms go to zero in the limit? $\lim_{k\to\infty} a_k = 0$?	NO	**Divergence Test**	$\lim_{k\to\infty} a_k \neq 0$	Series diverges
YES				
Is the series Geometric? $\sum_{k=0}^{\infty} a \cdot r^k$ $r = \frac{a_{k+1}}{a_k} = CONSTANT$	YES	**Geometric Series Test**	$\lvert r \rvert < 1$	Series converges to $\frac{a}{1-r}$
			$\lvert r \rvert \geq 1$	Series diverges
NO				
Is the series of the form $\sum_{k=1}^{\infty} \frac{1}{k^p}$	YES	**p-Series Test**	$p > 1$	Series converges
			$p \leq 1$	Series diverges
NO				
Is the series a rational expression made of two polynomials? EX: $\sum_{k=1}^{\infty} \frac{k^2+1}{k^3+4k+2}$ Compare to $\sum_{k=1}^{\infty} \frac{k^2}{k^3} = \sum_{k=1}^{\infty} \frac{1}{k}$	YES	**Direct Comparison** or	Series is OVER a divergent series	Series diverges
			Series is UNDER a convergent series	Series converges
		Limit Comparison In both cases, compare to a p-Series determined using "dominance"	$\lim_{k\to\infty} \frac{a_k}{b_k} = L$	Both series either converge or diverge…their terms' behavior is the same in the long run.
			$\lim_{k\to\infty} \frac{a_k}{b_k}$ *dne*	Test is inconclusive
NO (next page)				

Does the series have a factorial? EX: $\sum_{k=1}^{\infty} \frac{k!}{2^k}$	YES	**Ratio Test**	$\lim_{k\to\infty}\left\lvert\frac{a_{k+1}}{a_k}\right\rvert = L$	L<1, series converges
				L>1, series diverges
				L=1, test is inconclusive
Alternating Series have some further considerations				
Do the terms of $\sum_{k=1}^{\infty} a_k$ alternate in sign? EX: $\sum_{k=1}^{\infty} \frac{(-1)^k k^2}{3^k}$	YES	**First test for Absolute Convergence:** Does $\sum_{k=1}^{\infty} \lvert a_k \rvert$ converge or diverge? Apply one of the tests above.	$\sum_{k=1}^{\infty} \lvert a_k \rvert$ converges,	Series converges "absolutely"
			$\sum_{k=1}^{\infty} \lvert a_k \rvert$ diverges,	Check "conditional" convergence using Alternating Series Test
		Alternating Series Test $\lim_{k\to\infty} \lvert a_k \rvert = 0$? Note: You've already analyzed this in the first step (Divergence Test!)	YES	Series converges "conditionally"
			NO	Series diverges