

**Math 265B: Free Fall with Air Resistance**

**Name:** \_\_\_\_\_

You've just jumped off of a very high cliff. Sketch a curve that illustrates your velocity as a function of time. Include the steady state for this situation as part of your sketch.



Now, the air is resisting you as you fall. What type of physical quantity is the air resistance?

I.e., is it a mass? A distance? A force?

And the resistance is proportional to \_\_\_\_\_.

So  $F_{air} =$  \_\_\_\_\_, where  $k =$  \_\_\_\_\_

We can say that the two forces at play, then, are the force of \_\_\_\_\_

and the force of the \_\_\_\_\_.

Sketch a free-body diagram showing these forces. Note: We're going to let the positive direction be downward (so when you fall down, towards the Earth, your velocity will be positive).

Free body diagram:

Set up an equation to model the relationship and net effect of these forces:

$$\underline{\hspace{10em}} = \underline{\hspace{10em}} + \underline{\hspace{10em}}$$

*Net Force (think Newton's Second Law)*                      *Force of gravity*                      *Force of the air resistance*

Rewrite your equation here:

$$\frac{\text{Net Force (think Newton's Second Law)}}{\text{Net Force (think Newton's Second Law)}} = \frac{\text{Force of gravity}}{\text{Force of gravity}} + \frac{\text{Force of the air resistance}}{\text{Force of the air resistance}}$$

But what is  $a$  in terms of  $v$ ? \_\_\_\_\_ Substitute this into the equation, and voila!

We now have a Differential Equation in terms of  $v$ ! What is the initial condition?

$$\frac{\text{(Differential Equation)}}{\text{(Differential Equation)}} + \frac{\text{Initial Condition}}{\text{Initial Condition}}$$

Now solve for  $dv/dt$  and using this equation, find the equilibrium solution:

Graph the equilibrium solution on the graph on the first page.

Now fill in the slope field above and below the equilibrium solution. Your original concept sketch is now the solution curve to the I.V.P.

Solve the D.E. using Separation of Variables. Then solve the I.V.P. (evaluate  $C$  using the I.C.)

Follow up questions:

- (1) What happens to  $v(t)$  as  $t$  gets large? What is the name for this? Write it as a limit!
- (2) What would the solution curve look like if the I.C. were above the equilibrium line? Sketch this on the slope field graph. What would this mean in the real world?