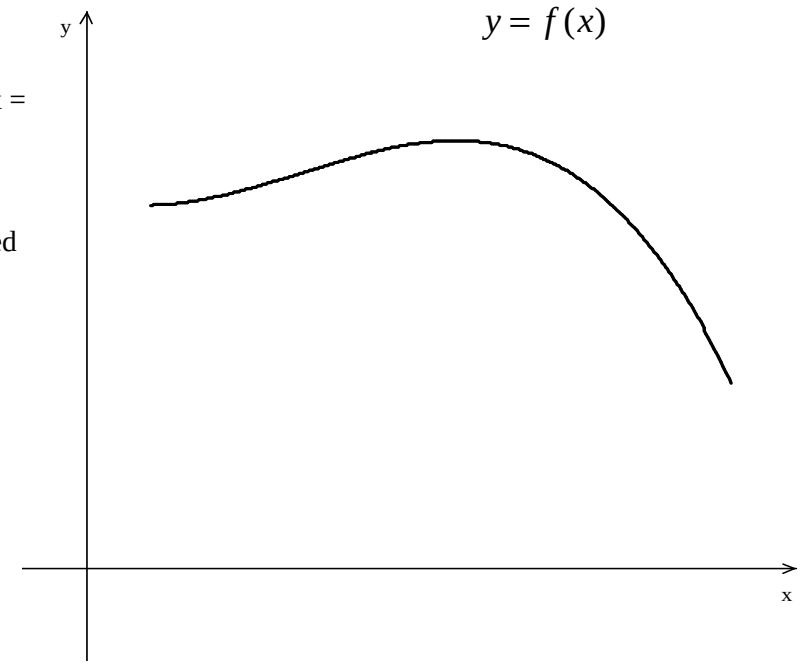
Math 265B: Deriving the Integral Formula for Arclength (Section 6.5)

Question: How long is the curve $y = f(x)$ from $x = a$ to $x = b$?

Approach: Slice, dice, and sum up the pieces!



1. <u>On the x-axis</u>: Label the location of the endpoints of the curve $y = f(x)$ with x = a and x = b, then partition the interval [a,b] into 5 or 6 subintervals.

<u>On the curve</u>: Use the points on the curve (created by the partition) to make a "polygonal approximation" to the curve.

2. Select one of the middle subintervals. We'll treat this as the k^{th} subinterval. Label the left endpoint <u>on the x-axis</u> as x_k .

<u>On the piece of the curve</u> in your k^{th} subinterval, draw a secant line, then draw a right triangle and label the parts as $\Delta x,\ \Delta y_k,\ L_k$,
where L_k = hypotenuse.

3. Relate $\Delta x,\ \Delta y_k,\ L_k$ with an appropriate equation. $\longrightarrow$

4. The crux: Write a <u>linear approximation</u> for Δy_k and substitute this into your equation above. (Use the derivative to make the linear approximation!) $\longrightarrow$

5. Solve for L_k $\longrightarrow$

6. Construct a Riemann sum for the total length of the curve (approximate) using L_k from above. Be sure to factor out Δx $\longrightarrow$

7. Conclusion : Use the Riemann sum to set up the integral that gives the exact length of the curve from a to b.

The length of the curve $y = f(x)$ on [a,b] = _____