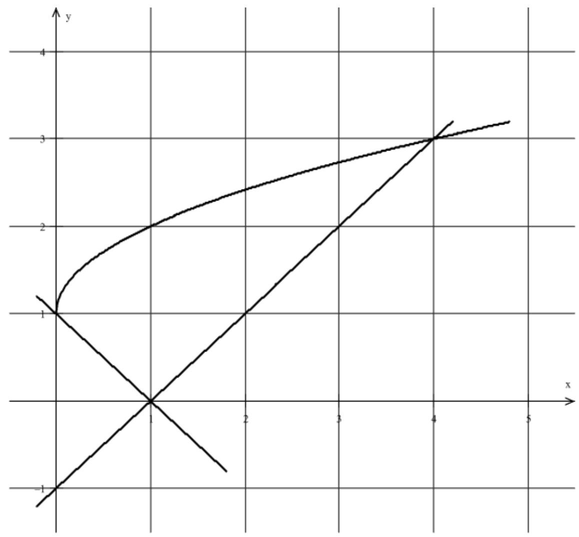
Math 265B: Final Exam **Name:________________________________**

(125 pts)

*Important note: Since many of these problems are possible to solve using technology, be sure that you are showing **clear and complete work**. Credit is based on the amount of correct work shown, not just the final answer, and correct answers without the correct work to support them will receive no credit at all. Give exact answers (leave pi as pi, etc.) unless an approximation is asked for.*

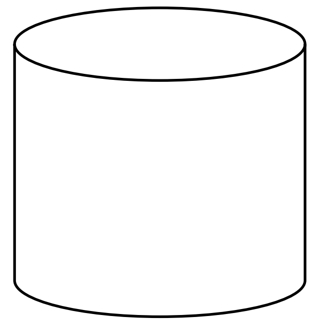
1. (4 pts) Consider the region bounded by $y = 1 + \sqrt{x}$, $y = x - 1$, and $y = -x + 1$, *as shown*. Use calculus to find the area of the region. Express the answer as a fraction, not a decimal.



2. (6 pts) Find the volume of the solid that is formed by revolving the region bounded by $y = \sqrt{x}$ and $y = x$ about the x-axis. Include a sketch to show your reasoning.

3. (6 pts) A buried cylindrical tank (see picture) has its top 8 feet beneath ground level. The tank has a radius 4 feet and is 6 feet high and is half-full of oil. (Oil weighs 50 lbs/ft^3.)

Use calculus to find the work done against gravity in pumping all of the oil to the surface.
Express your answer in terms of ρ, g, π , then give an approximation including units.



4. (5 pts) Determine which of the integration techniques listed below would be the *BEST* to apply to find the following integrals. You do not have to integrate.

Techniques: ***Partial Fraction Decomposition,*** ***Integration by Parts,*** ***u-Substitution ,***

Trig Substitution, ***Half Angle Trig Identity (Power Reduction),*** ***Pythagorean Trig Identify***

a.) $\int x\sin(x)\,dx$ Best Technique:________________________________

b.) $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$ Best Technique:________________________________

c.) $\int \frac{1}{x^3+5x}\,dx$ Best Technique:________________________________

d.) $\int \cos^2(x)\sin^2(x)dx$ Best Technique:________________________________

e.) $\int \frac{1}{\sqrt{x^2-25}}dx$ Best Technique:________________________________

5. (5 pts) Integrate. Show complete work for credit, and simplify as much as possible

$$\int_1^2 x^2 \ln(x)\, dx .$$

6. (7 pts) Integrate using Trig Substitution. $\displaystyle\int_1^2 \frac{\sqrt{4-x^2}}{x^2}dx$

7. (6 points) The given integral is improper. What makes it improper?

$$\int_0^2 \frac{1}{(x-1)^{4/3}}\,dx$$

Evaluate the integral and state whether it converges or diverges. Use proper limit notation and include a graph to illustrate your work.

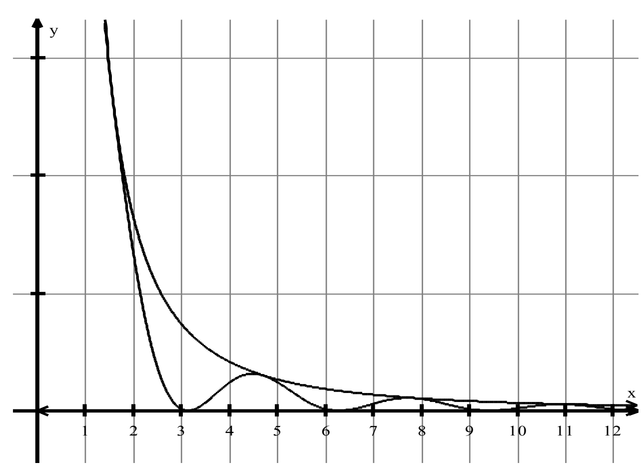
8. (6 pts) (a) Evaluate the integral and state whether it converges or diverges. Use proper limit notation.

$$\int_\pi^\infty \frac{1}{x^2}\,dx$$

(b) Based on the result from part (a), and the given graph $\frac{1}{x^2}$ and $\frac{\sin^2(x)}{x^2}$,

explain how you can tell whether the integral $\int_\pi^\infty \frac{\sin^2(x)}{x^2}\,dx$

converges or diverges.



9. (4 pts) If *f(x)* is positive, increasing, and concave down on [a,b] then which of these is a possible ordering of the approximations for $\int_a^b f(x)dx$? Sketch a graph of the integral and the approximations to explain your choice, using n = 2. A bigger picture is easiest to reason with.

a. LEFT(n) $\le$ TRAP(n) $\le$ RIGHT(n)   Picture:

b. RIGHT(n) $\le$ TRAP(n) $\le$ LEFT(n)

c. RIGHT(n) $\le$ LEFT(n) $\le$ TRAP(n)

d. LEFT(n) $\le$ RIGHT(n) $\le$ TRAP(n)

10. (2 pts) Determine whether each of the following series converges or diverges. Give a brief reason for each. (Do not do any formal testing here.)

(a) $\sum_{k=1}^{\infty} \left(\frac{3}{5}\right)^k$ converges / diverges (circle one) because ____.

(b) $\sum_{k=1}^{\infty} \frac{1}{k^{1/2}}$ converges / diverges (circle one) because ____________________________________

11. (6 pts) Find the interval of convergence for this series. Include determining whether or not to include the endpoints of the interval. $\sum_{k=0}^{\infty} \frac{x^k}{2^k}$

12. (3 pts) Derive the first three non-zero terms of the Taylor Series formula for cos(x).

13. (9 pts) Suppose a particle is moving along the path described by the parameterization $\begin{cases} x = 2 - t^2 \\ y = 3t - 1 \end{cases} \quad 0 \le t \le 3$

(a) Set up and fill in an appropriate table to graph the curve, then graph the curve. Show the orientation (direction of travel) and label each point with it corresponding t-value.

(b) Find the equation of the tangent line at the point where t = 2. Sketch the line on your graph from part (a).

(Extra credit (2 pts): Find the parametric form of the tangent line equation, including the appropriate time shift to ensure the particle is at the point of tangency at the right time.)

14. (6 pts) Find a parameterization of each of the following curves. Be sure to include the domain!

(a) The line segment that begins at point P(2, -1) and ends and point Q(-1,3)

(b) A circle of radius 4, oriented counterclockwise, with center at the point (1, 3) and period of 6 seconds.

15. (6 pts) Convert each of the xy-equations into a polar equation. Solve for r, if possible.

(a) $x^2 + y^2 = 6y$ (b) $y = 5x$ (c) $y = 5$

16. (14 pts) Consider the region, defined using polar coordinates, where the area inside the curve $r = \cos\theta$ overlaps the area inside curve $r = 1 - \cos\theta$ (so the area that is common to both).

a) Sketch a graph of the curves and shade the region as described. You don't have to show work for the graphs (i.e., using Desmos if fine).

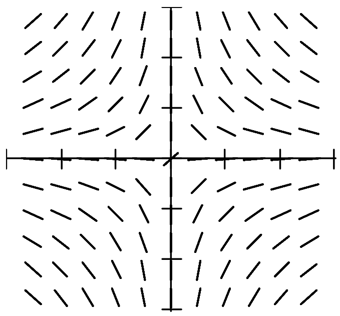
b) Find two points of intersection using algebra and trig. Confirm you work and find the third point using a graph. (Extra credit (2 pts): Explain why the third point can't be found algebraically.

c) Find the area of the shaded region, using calculus.

17. (3 pts) Determine which slope field matches the given differential equation. The window is [-3,3] by [-3, 3] in each graph.

$$\frac{dy}{dx} = \frac{1}{x}$$ Answer: ____________

I



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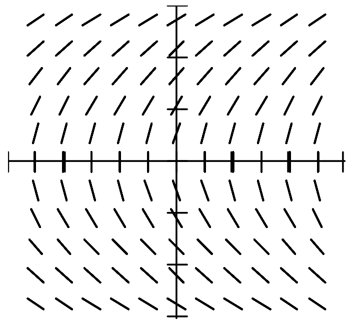
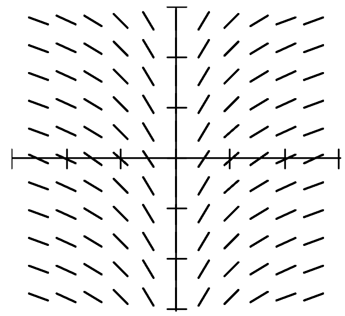


III



18. (6 pts) The Initial Value Problem given below is an example of the type of DE's you see in logistic models. Find the solution to the IVP in <u>explicit</u> form.

$$\frac{dy}{dt} = \frac{1}{y(5-y)} \quad y(0) = 2 .$$

Extra credit (3 pts): Use calculus to find the inflection point of the solution curve. Why is the inflection point important for logistic models of real situations, like epidemics?

11. (14 pts) A cup of coffee cools at a rate proportional to the difference between the coffee's temperature, H, and the ambient temperature, A. Suppose you're outside where the temperature is $60^{\mathrm{O}}F$ with a freshly-made cup of coffee that is $170^{\mathrm{O}}F$.

(a) Set up the differential equation and initial condition that model this situation.

(b) Is the differential equation Autonomous? How can you tell?

(c) <u>Find the equlibrium solution</u> (show work) using your DE and <u>sketch a slope field</u> that shows the equilibrium solution and <u>the solution curve</u> for the IC you found in part (a).

(d) Solve the IVP using Separation of Variables (show all steps).

(#11 continued)

(e) If the coffee cooled to be $115^{O} F$ after 5 minutes, what is the value of k? (Note: this will take some Precalculus algebra to solve. See Section 8.5 [9.5] for an example showing this.)

(Extra credit (2 pts): Explain what the physical meaning is of k, including units, in the context of this problem.

7. (7 pts) Consider the I.V.P. $\dfrac{dy}{dx} = x^2 - y, \qquad y(0) = 1$

(a) Use Euler's Method, with $\Delta x = 0.5$ to estimate the value of $y(x)$, for x = 1.5 Clearly show all of your work.

$y(1.5) \cong$ ________________

(b) On the slope field provided, sketch the polygonal Euler's solution "curve", based on your work from part (a).