Math 265B: Improper Integrals (Section 7.8)

Consider the definite integrals $\int_0^\infty \frac{0.5x}{x^2+1}dx$ and $\int_0^\infty \frac{1}{x^2+1}dx$.

1. Sketch two separate graphs to illustrate what each integral represents.

2. Since we cannot integrate to infinity, we will consider integrals on the interval [0, *b*] and observe what happens as we let *b* increase. Use Wolfram Alpha or another program to evaluate the integrals.

b	$\int_0^b \frac{0.5x}{x^2+1}dx$	$\int_0^b \frac{1}{x^2+1}dx$
10		
20		
50		
100		
250		

What do these results suggest about the two areas?

3. What is improper about the definite integral $\int_1^3 \frac{1}{(x-2)^2}\,dx$?

Sketch a graph showing the area represented by the integral. Is this region closed?

4. Pretend that you were unaware of the "improperness" of the integral in Question 3 and evaluate it like normal. Why is your answer clearly incorrect? (Consider the graph)

5. Let's consider integrals on the interval [1, *b*] and observe what happens as we let *b* get close to 2.

b	$\int_1^b \frac{1}{(x-2)^2}\,dx$
1.5	
1.9	
1.99	
1.999	

What do these results suggest about the area?