Math 265B: Integration by Parts, LIATE, and Column Method

Integration by Parts Formula: $\int u \cdot dv = u \cdot v - \int v \cdot du$

The expressions *u* and *dv* are chosen with two criteria in mind:
(1) *v* should be easy to find from *dv*; (2) the integral $\int v \cdot du$ should, in some sense, be "better" or easier than $\int u \cdot dv$

The difficulty usually arises in choosing *u* and *dv* to satisfy these conditions. An acronym gives us a method of selection which helps to satisfy criterion (2) above. The word is **LIATE**, standing for which function should be chosen as u.
Logarithmic **I**nverse-Trig **A**lgebraic **T**rig **E**xponential.

Integrate each of the following, using Integration by Parts Formula, with LIATE as your guide for choosing u

$\int xe^x \, dx$

$\int \sin^{-1}(x) dx$

$\int x^4 \ln(x) dx$

Round Robin:

$\int e^x \sin(x) dx$

Column Method:

If u can be differentiated down to zero and dv can easily be integrated repeatedly, then using the Column Method is the quickest way to go!

EXAMPLE 1 Evaluate $\int x^3 e^x \, dx$ by tabular integration.

Solution With $u = x^3$ and $dv = e^x$, we list

u and its derivatives		dv and its antiderivatives
x^3	(+) →	e^x
$3x^2$	(−) →	e^x
$6x$	(+) →	e^x
6	(−) →	e^x
0		e^x

We add the products of the functions connected by the arrow, with every other sign changed, to obtain

$$\int x^3 e^x \, dx = x^3 e^x - 3x^2 e^x + 6xe^x - 6e^x + C .$$

EXAMPLE 2 Evaluate $\int x^3 \sin x \, dx$ by tabular integration.

Solution With $u = x^3$ and $v' = \sin x$, we list

u and its derivatives		v' and its antiderivatives
x^3	(+) →	$\sin x$
$3x^2$	(−) →	$-\cos x$
$6x$	(+) →	$-\sin x$
6	(−) →	$\cos x$
0		$\sin x$

We add the products of the functions connected by the arrow, with every other sign changed, to obtain

$$\int x^3 \sin x \, dx = -x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6 \sin x + C .$$