Math 265B: Techniques of Integration...when the integrand is a fraction

Note: This is not a sequential or algorithmic approach. In dealing with more advanced integration, you have to have a pretty high tolerance for frustration and the willingness to try something to see where it leads, even if it might lead to a dead end.

Look first for a u-substitution (you're looking for u and du). Even if there isn't a u-sub at first, *keep looking for one* as you continue with the other strategies described below!

Forms: $\int \frac{1}{u}\,du = \ln|u| + C$

or $\int \frac{1}{u^p}\,du = \int u^{-p}du = \frac{1}{-p+1}u^{-p+1} + C$, where $p \neq 1$

Look for integrands which have the form of inverse trig function derivatives.
You may uncover this pattern *after* doing u-substitution!

$\int \frac{1}{1+u^2}\,du = \tan^{-1}(u) + C$ $\int \frac{1}{\sqrt{1-u^2}}\,du = \sin^{-1}(u) + C$ $\int \frac{1}{u\sqrt{u^2-1}}\,du = \sec^{-1}(u) + C$

Manipulate the integrand algebraically. Some methods that can help:

- Try breaking up the fraction (split up the numerator).

  Example: $\int \frac{x+5}{\sqrt{1-x^2}}\,dx$

- Try applying trig identities or try writing all functions in terms of sine and cosine

  Example: $\int \frac{\sin^2(x)}{\sqrt{1-\cos(2x)}}\, dx$

- Try multiplying the fraction by ***a form of one*** to massage the integrand into an integrable form.

  Example: $\int \frac{e^{-2x}}{e^{-x}+e^{-3x}}\, dx$

- If the integrand is a rational function with the degree of numerator $\geq$ degree of denominator, then perform long division.

  Example: $\int \frac{x^2}{x+1}\, dx$