Math 265B: Integration Review (Chapter 5)

Anti-derivatives/Indefinite Integral Formulas

Power rule:

$$\int u^p du = \frac{1}{p+1} u^{p+1} + C$$

Natural Log:

$$\int \frac{1}{u} du = \ln|u| + C$$

Exponential Functions:

$$\int e^u du = e^u + C$$

$$\int a^u du = \tfrac{1}{\ln(a)} a^u + C$$

Substitution (Reversing the Chain Rule:

$$\int f\left(u(x)\right) \cdot u'(x)\ dx = F\left(u(x)\right) + C$$

where F is an antiderivative of f.

Method: **First** try Guess and Check! Compensate with a multiplied constant, if necessary, to make sure that $\frac{d}{dx}\left[F\left(u(x)\right)\right]$ matches the integrand.

Second, if you can't guess correctly, look for an "inner function" (this will be "u") and the derivative of that function (this will be "du"). Compensate with any needed <u>constant</u> to make du precise.

Trig Functions:

$$\int \cos(\mathrm{u}) du = \sin(\mathrm{u}) + C$$

$$\int \sin(\mathrm{u}) du = -\cos(\mathrm{u}) + C$$

$$\int \sec^2(\mathrm{u}) du = \tan(\mathrm{u}) + C$$

$$\int \sec(\mathrm{u}) \tan(\mathrm{u})\ du = \sec(\mathrm{u}) + C$$

Inverse Trig Functions:

$$\int \frac{1}{u^2+1} du = \arctan(\mathrm{u}) + C$$

$$\int \frac{1}{\sqrt{1-u^2}} du = \arcsin(\mathrm{u}) + C$$

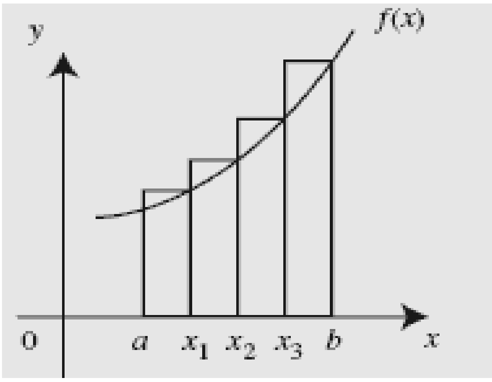
$$\int \frac{1}{u\sqrt{u^2-1}} du = \mathrm{arc}\sec(\mathrm{u}) + C$$

Riemann Sums: (Note: we'll be revisiting these in Chapter 7. For now, we'll just review the Right Sum)

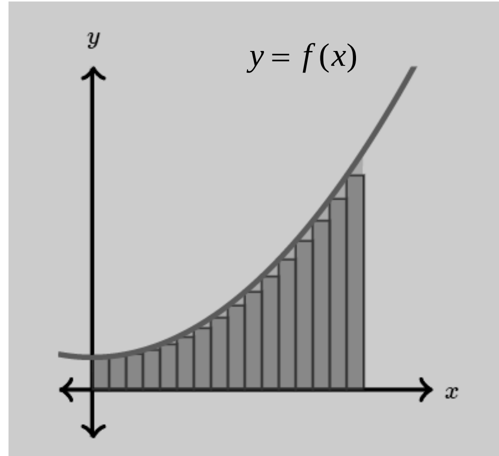
Right Sum: $\sum_{k=1}^{n} f(x_k)\Delta x$,

where n = the number of subdivisions of the interval [a, b]

and $\Delta x = \frac{b-a}{n}$



Definite Integrals: Suppose *f* is continuous on the interval [a,b] .
We define the Definite Integral of *f* on the interval [a,b] to be



$$\int_a^b f(x)dx = \lim_{\Delta x \to 0} \sum_{k=1}^{n} f(x_k)\ \Delta x$$

or more commonly, $\int_a^b f(x)dx = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k)\ \Delta x$

Fundamental Theorem of Calculus: If *f* is continuous on the interval [a,b]

$\int_a^b f(x)dx = F(x)\Big|_{x=a}^{x=b} = F(b) - F(a)$, where F is any antiderivative of *f* , i.e., $F'(x) = f(x)$

Using symmetry to evaluate integrals:

$\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$ if *f* is an <u>even</u> function, i.e. symmetric with respect to the y-axis.

Test for even function: $f(-x) = f(x)$

$\int_{-a}^{a} f(x)dx = 0$ if *f* is an <u>odd</u> function; i.e. symmetric with respect to the origin.

Test for odd function: $f(-x) = -f(x)$

Properties of Integrals:

1. $\int_a^a f(x)dx = 0$
2. $\int_b^a f(x)dx = -\int_a^b f(x)dx$
3. $\int_a^b f(x) + g(x)dx = \int_a^b f(x)dx + \int_a^b g(x)dx$
4. $\int_a^b cf(x)dx = c\int_a^b f(x)dx$
5. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

Average value of a function over an interval [a,b]: $f_{avg} = \dfrac{1}{b-a}\int_a^b f(x)dx$