Math 265B: Summary of Parametric Equations

Parameterization: A "parameterization" of a path in 2-space is a set of equations where x and y are functions of t. The variable "t" is called the "parameter". It is the independent variable while x and y are both dependent (on t).

Any parameterization *<u>must also include a domain for t</u>*.

The parameterization of a path is generally <u>not unique</u>; i.e., there are many parameterizations that will all describe the same path.

General advice: After setting up a parameterization, ALWAYS check that it "works", i.e., that it begins at the correct point and goes in the correct direction to get to the next point.

To graph a parameterization, you can use a table:

Given $x = x(t), \quad y = y(t), \quad t_0 \le t \le t_1$ Set up and fill in a table, like this:

t		x	y

<u>To reverse direction</u>: In general: Substitute $-t$ for t in the parametric equations and simplify.

Circles and Ellipses:

Unit Circle

No parameter: $x^2 + y^2 = 1$ (Boring and static…)

Parametric form: $x = \cos(t), \quad y = \sin(t), \quad 0 \le t \le 2\pi$ (Dynamic!)

Includes direction of travel and period, which relates to speed)

Direction: **counter-clockwise** (ccw) ("Direction" is also called "Orientation")

Period: $T = 2\pi$

If direction of motion is **clockwise**: $x = \cos(t), \quad y = -\sin(t), \quad 0 \le t \le 2\pi$

Circles in General: Center (h,k), Radius a

No parameter: $(x-h)^2 + (y-k)^2 = a^2$

Parametric form: $x = h + a\cos(bt), \quad y = k + a\sin(bt), \quad 0 \le t \le \frac{2\pi}{b}$

(ccw-orientation)

Period: $T = \dfrac{2\pi}{b}$

Ellipses:

No parameter: $\dfrac{(x-h)^2}{A^2} + \dfrac{(y-k)^2}{B^2} = 1$

Parametric form: $x = h + A\cos(t), \quad y = k + B\sin(t), \quad 0 \le t \le 2\pi$

(ccw-orientation)

Period: $T = 2\pi$

Lines: Constant velocity lines

Parametric form (<u>ENTIRE LINE</u>): $x = x_0 + at, \quad y = y_0 + bt, \quad -\infty < t < \infty$

Parametric form (<u>LINE SEGMENT</u>): $x = x_0 + at, \quad y = y_0 + bt, \quad t_0 \le t \le t_1$

Slope of this line: $m = \frac{b}{a}$

To find the parameterization of a line or line segment:

If the line passes through a point (x_0, y_0) and has slope $m = \frac{\Delta y}{\Delta x}$,
a parametric form of the ENTIRE LINE is $x = x_0 + \Delta x \cdot t, \quad y = y_0 + \Delta y \cdot t, \quad -\infty < t < \infty$

If you just want the parameterization of the line segment $\overline{PQ}$ that goes from point $P = (x_0, y_0)$ to point $Q = (x_1, y_1)$ in one time increment, then do the following.

Find $\Delta x = x_1 - x_0, \quad \Delta y = y_1 - y_0$

Parameterization is $x = x_0 + \Delta x \cdot t, \quad y = y_0 + \Delta y \cdot t, \quad 0 \le t \le 1$

Parametric Equations and Motion:

Velocity: $v_x = \frac{dx}{dt}$ is the velocity in the x-direction,

$v_y = \frac{dy}{dt}$ is the velocity in the y-direction

Speed: $s = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$ is the speed.

Acceleration: $a_x = \frac{d^2x}{dt^2}$ is the acceleration in the x-direction

$a_y = \frac{d^2y}{dt^2}$ is the acceleration in the y-direction